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Growth, biomass and stem volume models for downy birch in Iceland

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ABSTRACT
Downy birch (*Betula pubescens* Ehrh) is the only native tree species forming woodland in Iceland and covers around 1.5% of the land area. Historically, birch woodlands have been an important source of fuel, building material and animal fodder. Currently, birch wood production is not economically important, but the species provides other valuable ecosystem services, such as soil conservation and carbon sequestration. Planting of birch increased in the 1990´s, and from 2000 onwards the share of birch has been around 30% of the total annual tree planting. Growth models are essential tools for simulating forest development over time and are widely used in forest management, research, and policymaking. This study presents site index, individual-tree diameter (dbh 1.3 m) increment, and tree height models for even-aged stands of cultivated downy birch in Iceland. The data were collected from the Icelandic National Forest Inventory (NFI) plots measured between 2005 and 2024. In addition, models for aboveground biomass and stem volume were developed using dbh as a predictor. The models can be used in forestry practice and the management optimization of downy birch stands.

Keywords: Birch, growth model, aboveground biomass, stem volume model, simulation.

YFIRLIT
*Jöfnur sem lýsa vexti vexti birkis (*Betula pubescens Ehrh*) á Íslandi*
Birki (*Betula pubescens* Ehrh) er eina innlenda trjátegundin sem myndar skóga á Íslandi og þekur um 1,5% af yfirborði Íslands. Í gegnum tíðina hafa birkiskógar verið verðmæt náttúruauðlind sem eldsneyti, byggingarefni og fóður, en nú til dags eru viðarafurðir birkis ekki efnahagslega mikilvægar. Vistþjónusta birkiskóga og birkiskógræktar er samt mikilvæg t.d. við jarðvegsvernd og kolefnisbindingu. Skógrækt með birki jókst mjög á síðasta áratug seinustu aldar og frá upphafi þessarar aldar hefur hlutfallslegur fjöldi gróðursettra birkitrjáa verið um 30% af árlegri gróðursetningu. Vaxtarlíkön spá fyrir um framtíðarvöxt skóga og eru mikilvæg verkfæri fyrir skógarstjórnendur, vísindamenn og við stefnumótun, meðal annars í loftslagsmálum. Í þessari rannsókn eru birt líkön sem lýsa gróskustigi, þvermálsvexti og hæðarvexti trjáa fyrir jafnaldra ræktaðra birkiskóga á Íslandi. Gögnin sem notuð voru í rannsókninni eru trjámælingar frá Íslenskri skógarúttekt sem gerðar voru á árunum 2005 til 2024. Einnig var þróað líkan til að spá fyrir um heildarlífmassa og heildarrúmmál trjábols sem nota þvermál í brjósthæð (1,3 m). Líkönin má nota til áætlanagerðar og til að spá fyrir um vöxt og kolefnisbindingu í lífmassa í birkiskógum.

INTRODUCTION

Downy birch (*Betula pubescens* Ehrh) is the only native tree species forming large woodlands and forest stands in Iceland (Aradottir & Eysteinsson 2005). It covers around 1.5% of Iceland's land area (Snorrason et al. 2016). Its natural growth form is often of low stature and crooked stems, resembling the subarctic mountainous downy birch that is common at the tree line in Scandinavia, often named mountain birch (Snorrason et al. 2016). The downy birch in Iceland is often hybridized with dwarf birch (*B. nana* L.), a small shrub birch species, which, to some extent, explains its low height and crooked stems (Anamthawat-Jónsson et al. 2021).

At the time of human settlement in the 9th century AD, natural birch woodland and forests are estimated to have been the dominant ecosystem across the Icelandic lowlands, covering most mineral soil types and approximately 20–30% of the country's terrestrial area (Snorrason et al. 2016). Historical evidence shows that the anthropogenic destruction of the woodland took place soon after the settlement, as birch forests were cut down or burned to give space for grazing. Throughout history, birch woodlands were also valuable for fuel, building materials, and animal fodder. Charcoal from woodlands was used for iron works, and the forests were extensively grazed (Thorarinsson 1974). Birch woodlands probably reached their postglacial minimum at the beginning of the 20th century, with about 1% cover of the total land area. A legislation issued in 1907 aimed at protecting the remaining woodlands and promoting new forests (Aradottir & Eysteinsson 2005). The purpose was to prevent further degradation and enhance the restoration of the woodlands. In many cases, this led to the expansion of birch forests (Blöndal and Gunnarsson 1999).

Active afforestation, involving planting and direct seeding of downy birch, began in 1899 but remained very limited until after the Second World War (Pétursson 1999). Although native birch is currently one of the five most planted species in Icelandic forestry, it was not until the 1990s that the annual number of planted native birch seedlings exceeded one million (Pétursson 1999). Since the beginning of the current century, the share of birch among all planted seedlings has been around 30%. According to official reports for 2023, birch was the most planted species, with a share of 32% (Jóhannsdóttir et al. 2024). Today, birch wood products are not economically significant, although a small amount (less than 500 m^3) is harvested each year and sold for firewood and handicrafts (Aradottir & Eysteinsson 2005). Its utilization will likely increase when the birch plantations mature.

The most widely used birch provenance in plantation forestry in Iceland is Bæjarstaðarbirki, along with Bæjarstaðarúrval, both originating from a natural high-stature forest in Southeast Iceland. These provenances show good growth and superior stem form compared with many other Icelandic birch provenances. Bæjarstaðarúrval refers to a selection of 40 individual trees chosen from the Bæjarstaður forest in 1994 for seed production (Eysteinsson et al. 2023).

In recent years, a forest planning system for sustainable forest management has been under development in Iceland. The system is called IceForest, and it is a modification of the Finnish Monsu system (Pukkala 2004, Heinonen et al. 2018, Díaz-Yáñez et al. 2020). The main steps and tools in planning are inventory, data management, simulation of stand development under alternative management schedules, and the selection of the optimal combination of the management schedules for the stands or other calculation units (Pukkala 2002).

Generally, growth models refer to a system of models that can predict the growth and yield of a forest stand under a wide variety of conditions (Vanclay 1994). Growth models can broadly be classified as stand-level or tree-level models. Stand-level models use stand variables (e.g., age, site index [SI], basal area [BA] and number of trees per hectare) as predictors, while at least some of the predictor variables in a tree-level model are individual tree characteristics (Clutter et al. 1983, Palahí et al. 2003, Weiskittel 2011). When individual-tree information for a stand is available, tree-level models should be used because they provide more detailed information on the stand structure and its dynamics than stand-level models (Mabvurira & Miina 2002, Palahí et al. 2003, Juma et al. 2014). The recent trend in Iceland has been to develop distance-independent, individual-tree models (Heiðarsson & Pukkala 2012, Heiðarsson et al. 2022, 2023, 2024). These models were selected because, compared to distance-dependent models, they are simpler to implement in operational forestry and require fewer data inputs. Distance-dependent models have not been shown to substantially improve predictive performance under typical stand conditions (Vanclay 1994, Pretzsch 2009, Burkhart & Tomé 2012).

Growth models typically predict the increment of the diameter at breast height (dbh), which is the most widely used variable in growth, biomass and volume calculations (e.g., Vanclay 1994, Burkhart & Tomé 2012). Stand management decisions, such as when and how much to thin a stand, rely heavily on variables derived from tree dbh and predicted dbh increment (Vanclay 1994, Pretzsch 2009, Burkhart & Tomé 2012). Information on dbh increment can be obtained from repeated measurements of permanent sample plots. There is a reasonable number of repeated measurements available on pure birch plots for various regions of Iceland, making it possible to develop different growth models for the species.

For even-aged monocultures, SI models are the most common tools for estimating site productivity. SI is defined as the dominant height, i.e., the average height of the 100 largest trees per hectare, at a chosen reference age (Monserud 1984, Skovsgaard & Vancley 2008, Burkhart & Tomé 2012). For most tree species, the height growth of dominant and co-dominant trees in a stand is a stable predictor of site quality because it is not much affected by stand density and thinning operations, assuming thinning from below (Cieszewski & Bella 1989, Skovsgaard & Vancley 2008, Burkhart & Tomé 2012). Information on tree heights is also essential in forest inventories for accurately computing tree volumes and biomass (Mehtätalo et al. 2015). Because field measurements of tree height are rather time-consuming and therefore expensive, many forest inventories use models to predict tree height from dbh.

In the present study, we developed distance-independent, individual-tree models for downy birch plantations. The model set consists of a site index (SI) model (top height growth model), a tree height model, and an individual tree model for diameter increment. Because the Icelandic datasets for birch forests include, in most cases, diameter measured at 0.5 m above ground ($d_{0.5}$) instead of dbh (Snorrason & Einarsson 2006, Snorrason 2010), a linear conversion model was also developed to enable the use of historical or mixed data sources. This allowed us to integrate existing measurements into modeling and simulation. In addition, new models for total aboveground biomass and stem volume were developed for estimating the yield and carbon stock of the biomass of birch plantations.

MATERIAL AND METHODS

The data used for modelling the growth of downy birch stands were collected at the permanent plots of the Icelandic National Forest Inventory (NFI) during 2005–2024. The database included 12,661 trees from 243 plots. The plots were remeasured at 5-year intervals. The NFI data are a statistical sample of all forested land areas in Iceland (Snorrason 2010). The plots used in this study were all planted, pure, even-aged birch stands. The dataset covered different site types and growth conditions, and all the locations have an oceanic climate. The annual precipitation at 8 selected weather stations (1991–2020) ranged from 573 to 1730 mm, and the mean annual temperature was 4.1 to 5.8°C (Vedurstofa Islands 2025). For the same period, the mean maximum daytime temperature during June–August was 10.5–15.5 °C (Vedurstofa Islands 2025). The range in plot elevation was between 40 and 400 m a.s.l.

The sample plots were circular, and the size of the plots varied between 0.005 and 0.02 ha. Height (h) was measured with a measuring pole for trees shorter than 4 m and Laser Tech instruments for taller trees.

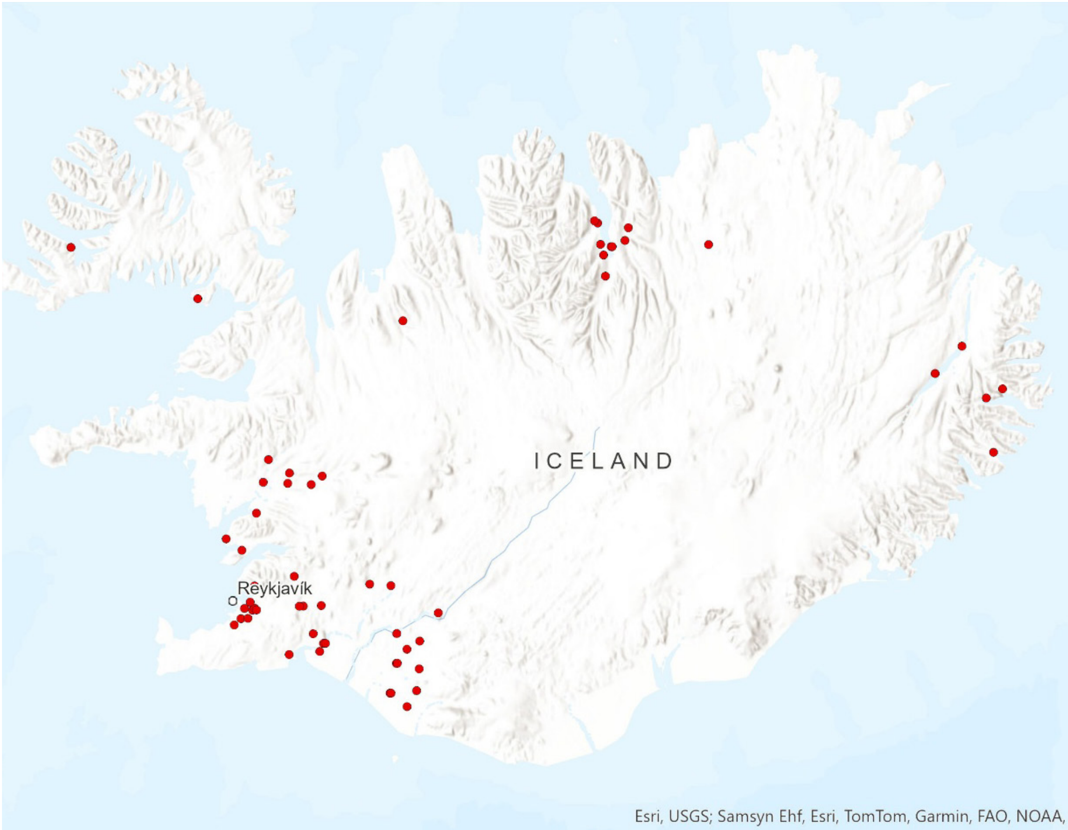

Figure 1. Geographical locations of the downy birch plantations in Iceland (red dots) where the material used for modelling height and diameter at breast height increment originated from.

Table 1. Mean, standard deviation (SD) and range of the main characteristics in the empirical data of the downy birch study material and number of observations (N).

Variable	N	Mean	SD	Maximum	Minimum
Diameter at breast height (dbh, cm)	2136	3.16	2.85	15.8	0.03
Height (h, m)	2023	3.01	1.55	10.49	1.3
Growth (dbh, cm)	2136	1.34	0.95	5.2	0.0
Stand basal area (BA, m^2ha^{-1})	89	4.67	8.11	48.7	0.006
Age (years)	89	23.02	10.61	60.0	7.0
Dominant height (H_{dom}, m)	160	4.14	1.95	10.5	1.3
Site index (SI, m)	160	6.67	1.80	11.14	2.15
Data for biomass modelling					
Above-ground biomass (kg DM $tree^{-1}$)	34	28.37	23.84	90.34	1.51
dbh (cm)	34	9.5	3.97	17.17	3.0
h (m)	34	6.5	1.65	10.3	3.7
Data for volume modelling					
Stem volume (dm^3 $tree^{-1}$)	46	34.79	31.47	122.3	0.55
dbh (cm)	46	8.97	4.15	17.15	1.09
h (m)	46	6.33	1.67	10.3	2.07

The number of measured heights was slightly lower than the dbh measurements (Table 1) because, in some plots, heights were not measured on all trees. The criteria for using the plot data for h and dbh increment modelling were that all tree heights were higher than 1.3 m. For SI modelling, a minimum age of 4 years was chosen because the height growth in very young stands is affected by factors other than site quality (Borders et al. 1984, Barrio Anta & Dieguez-Aranda 2005). All plots used in this study were unthinned.

Diameter conversion modelling

The tradition in Iceland is to measure the diameter at 0.5 m above ground on birches. Therefore, only part of the dataset included measurements at dbh. The management planning system used in Icelandic forestry uses dbh for all other species. Therefore, we developed a model for converting d0.5 to dbh. The available data consisted of 1625 trees for which both d0.5 and dbh were measured. The model is a simple linear model.

$$\hat{d} = a_0 \times d_{0.5} + a_1 \qquad (1)$$

where $d_{0.5}$ is the stem diameter at 0.5 meters from ground (cm), d is dbh (cm), and a_0 and a_1 are parameters.

In the national forest inventory data used in this study, stem diameter was not always measured in a fixed direction between measurement occasions, and measurement height was not permanently marked on individual trees. As a result, some additional measurement errors in diameter and diameter increment are expected, compared with datasets based on permanently marked measurement points or multi-directional diameter measurements.

Site index modelling

The data for model development were collected from 160 sample plots. Several models commonly used in the algebraic difference approach (ADA, Palahí et al. 2004) were tested. The tested models were those of: Korf and Lundmark (Korf 1939), Schumacher (1939), Chapman-Richards (Richards 1959) and McDill and Amateis (1992). All models predict the dominant height H_2, at a certain time point, T_2, using the current dominant height H_1 and current age, T_1, as predictors:

$$H_2 = f(T_1, H_1, T_2) + \mu + \varepsilon \qquad (2)$$

where μ is a random plot factor and ε is the residual. When T_1 is replaced by index age and H_1 is replaced by site index (dominant height at index age), the model gives the dominant height at age T_2 for site index H_1. If H_1 is the measured dominant height at age T_1, and T_2 is the index age, the model gives the site index.

The selected model was the model of McDill and Amateis (1992):

$$\widehat{H_2} = \frac{a_0}{1-\left(1-\frac{a_0}{H_1}\right)\times\left(\frac{T_1}{T_2}\right)^{a_1+u}} \qquad (3)$$

where H_1 and T_1 are, respectively, dominant height and stand age at the first measurement, H_2 and T_2 are the same variables at the second measurement, a_0 and a_1 are parameters to be estimated, and u is a random plot factor.

The model was fitted as a mixed-effects model. A random plot factor was added to a_0 or a_1, or both. The best combination of random plot factors was obtained by testing all combinations.

Tree height modelling

Based on the study of Mehtätalo et al. (2015), the following models were tested: Näslund (1937), Schumacher (1939) and Curtis (1967). These models were the best among the 28 datasets tested in that study. Among the tested models, the model of Schumacher (1939) turned out to be the most satisfactory.

$$\hat{h} = 1.3 + (a_0 + (a_1 + u_1)Hdom) \times \exp\left[\frac{-\{b_0+(b_1+u_2)Hdom\}}{d}\right] \qquad (4)$$

Where h is tree height, $Hdom$ is dominant height, and d is diameter at breast height. The parameters a_0, a_1, b_0, and b_1 are fixed effects describing the average dbh–height relationship across all plots, while u_1 and u_2 are random plot-level effects that allow this relationship to vary among plots. Specifically, the random effects modify the shape and steepness of the height–dbh curve by introducing plot-specific deviations in the influence of dominant height, thereby accounting for unobserved differences in stand structure and local growing conditions. The optimal random-effects structure was identified by testing all relevant parameter combinations. By expressing the model parameters as functions of dominant height, the dbh–height relationship is allowed to shift dynamically along stand development.

Diameter increment modelling

The data for dbh increment modelling were from 59 permanent plots, remeasured at 5-year intervals (Table 1). The data included 89 five-year growth periods. The longest series consisted of five measurements and four 5-year periods over 20 years. When dbh was not measured, and tree height was 1.3 m or higher, the new model to convert diameter at 0.5 m to dbh was used to estimate dbh. There was no mortality in the dataset, so no attempt was made to model tree survival.

The intention was to develop a model for the future 5-year dbh increment, which includes at least one predictor for each of the following three influences: tree size, competition, and site productivity. Tree size was described by dbh and its transformations, and the site index was used to describe the effect of site productivity. To describe competition, the stand basal area and the basal area in trees larger than the subject tree (BAL) were tested. Stand age was also tested, which described site productivity and the stage of stand development.

Regression analysis and mixed-effects modelling were used to search for the best transformations and combinations of predictors for the model. The following model turned out to be the most satisfactory:

$$\widehat{i_d} = \exp\left(a_0 + u_0 + a_1\sqrt{d} + a_2\left(\frac{d}{10}\right)^2 + (a_3 + u_1)\ln G + a_4 \ln SI + (a_5 + u_2)\left(\frac{BAL}{\sqrt{d+1}}\right)\right) \quad (5)$$

where i_d is the dbh increment (cm), d is the diameter at breast height (cm), G is BA (m² ha⁻¹), SI is the site index (m), and BAL is the basal area of trees larger than the subject tree (m² ha⁻¹). The parameters u_0, u_1, and u_2 are random plot-level effects representing variation among plots in overall growth level (random intercept) and in the response of diameter increment to stand density and competition (random slopes for ln$G$ and BAL).

The random-effects structure was selected by comparing alternative combinations of random intercepts and slopes using information criteria and variance component estimates, while also considering model convergence and parameter identifiability. This structure provided a favorable balance between model fit and parsimony, capturing meaningful between-plot variability without indications of overparameterization.

Stem volume and total aboveground biomass modelling

The new biomass and volume models use dbh instead of diameter at 0.5 m as the predictor. The data for the biomass and volume models were collected in a research project dealing with the growth potential of different tree species planted in Iceland, described in Snorrason & Einarsson (2006). The data consisted of 34 trees for total biomass and 46 trees for stem volume (Table 1).

Multiple nonlinear models evaluated in previous studies (Schumacher & Hall 1933, Spurr 1952, Clutter et al. 1983, Brandel 1990, Romero et al. 2020) were tested to model aboveground biomass and stem volume as functions of diameter at breast height (dbh) and tree height. Tree height was not a significant predictor of biomass and was therefore excluded from the biomass model (6). However, height was a significant predictor in the stem volume model and was retained in the final formulation (7). The selected models were:

$$\widehat{BM} = a_1 \times (1 + d)^{a_2} \quad (6)$$

$$V = b_1 \times (d^2 \times h)^{b_2} \quad (7)$$

where *BM* is the total aboveground biomass (kg), *V* is the total stem volume (dm^3) of the tree, *d* is the dbh, *h* is the tree height (m), and a_1, a_2, b_1, and b_2 are parameters. To eliminate the bias effect of the error variance for total biomass and stem volume models, which generally increases with tree size, and to reduce the effect of heteroscedasticity in the nonlinear models, the final model parameters were estimated with weighted nonlinear regression, where the weight was inversely proportional to d^4 ($1/d^4$). The biomass model (6) follows a power-function form commonly used in allometric biomass studies (e.g. Brandel 1990, Romero et al. 2020), whereas the volume model (7) is based on the classical combined-variable formulation originally proposed by Schumacher and Hall (1933).

All the models tested were evaluated and compared based on bias (*B*) for systematic errors, RMSE as an indication of precision, the degree of explained variance (R^2) and AIC. The model fit statistics were defined as:

$$B = \frac{\Sigma(y_i - \widehat{y_i})}{n} \quad (8)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}{n}} \quad (9)$$

$$R^2 = 1 - \left[\frac{\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}{\sum_{i=1}^{n}(y_i - \overline{y_i})^2}\right] \quad (10)$$

$$AIC = 2k - 2\ln(\hat{L}) \quad (11)$$

where y_i, $\hat{y}_i$ and $\bar{y}_i$ are the measured, predicted and average values of the dependent variable, *n* is the total number of observations used to fit the model, *k* is the number of estimated parameters and $\hat{L}$ is the maximized value of the likelihood function for the model.

For all models, the random error term was assumed to be independent and identically distributed with a mean equal to zero and constant variance. It was confirmed that the signs of all regression coefficients were consistent with the assumed influence of the different factors. For example, increasing competition is expected to decrease dbh growth, whereas improving site index should increase it. In addition, dbh increment is expected to decline at larger tree sizes as trees approach physiological and structural growth limits (e.g., Wykoff 1990, Weiskittel et al. 2011). Distributions of residuals were also examined for any biases. In the preliminary testing of model selection, the same statistics were used as in the final model selection. The estimated parameters for the new models were used to calculate fitting statistics.

All regression analyses were carried out with the R software, version 25.09.1 (Posit team 2025), using linear and nonlinear regression analysis.

RESULTS

Figure 2 illustrates key relationships among the variables used in the modelling. A strong positive relationship between tree height and dbh is evident (Figure 2A), supporting the use of dbh as a predictor of height when height measurements are unavailable. A clear negative relationship between basal area in larger trees (BAL) and 5-year dbh growth is shown in Figure 2B, indicating that increasing competition from larger neighboring trees is associated with reduced diameter growth.

Diameter conversion model

The parameter estimation for the model to convert stem diameter at 0.5 m to dbh was as follows:

$$\hat{d} = 0.851 \times d_{0.5} - 0.998 \quad (12)$$

where *d* is dbh and $d_{0.5}$ is the stem diameter at 0.5 m in cm. The model showed a strong linear relationship between $d_{0.5}$ and dbh (Figure 3A). The residuals were symmetrically distributed around zero and showed no major patterns or heteroscedasticity (Figure 3B). The residual standard error was 0.85, the explained variance (R^2) was 0.92, and the bias was <0.005, indicating a good model fit.

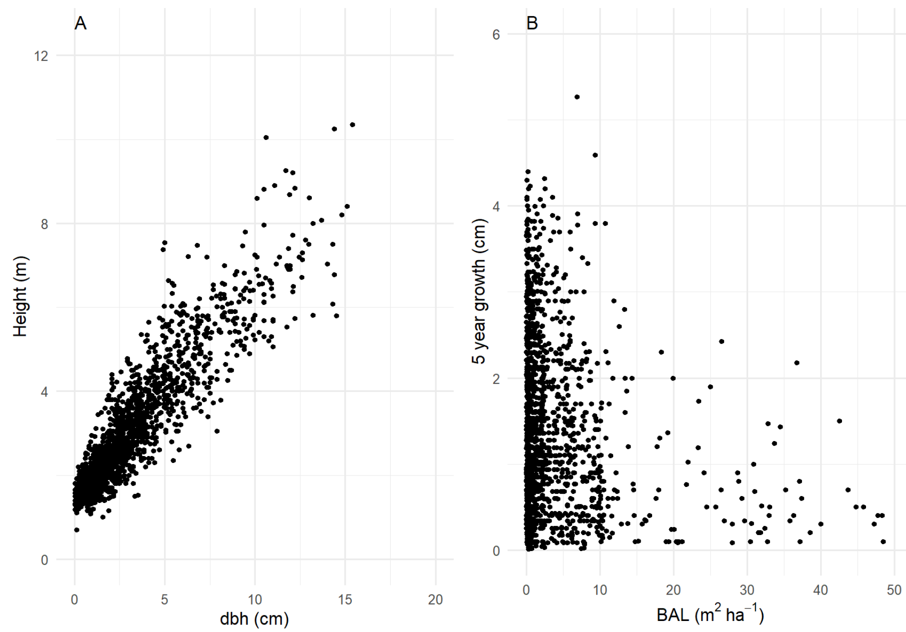

Figure 2. Observed A) diameter at breast height (dbh) and tree height, and B) basal area in larger trees (BAL) and 5-year dbh growth within the planted downy birch sample plots used in the study in Iceland.

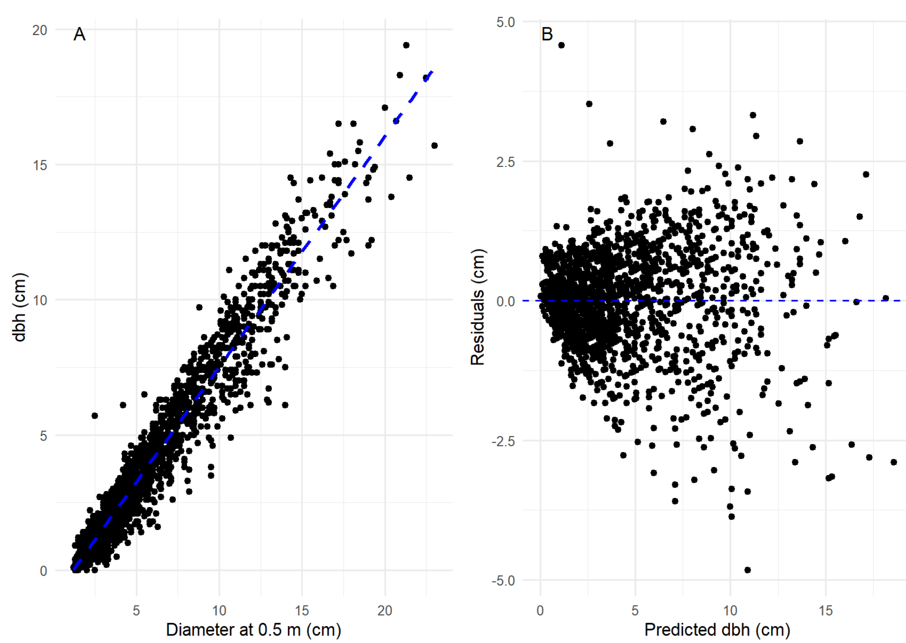

Figure 3. Relationships between: A) diameter measured at 0.5 m above ground (d0.5) and diameter at breast height (dbh) for planted downy birch based on measurements from permanent sample plots in Iceland. The dashed line indicates the fitted linear conversion model used to transform d0.5 to dbh. B) shows the residuals plotted against predicted dbh, indicating no systematic error across the observed diameter range.

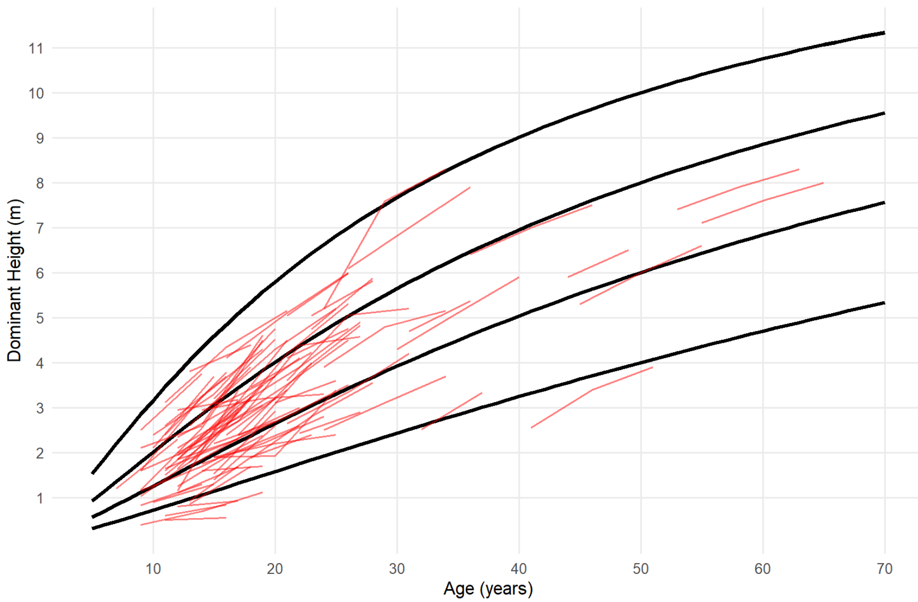

Figure 4. Site index curves for planted downy birch in Iceland (thick black lines) for site indices 10, 8, 6 and 4 m (site index = dominant height at 50 years) and the measured age and dominant height sequences of the study plots sampled around Iceland (red lines).

Site index model

Of the site index models tested, the model type of McDill and Amateis (1992) had the best fit statistics and a logical behaviour when extrapolating outside the data range, and was therefore selected:

$$\widehat{SI} = \frac{15.364}{1-\left(1-\frac{15.364}{H}\right)\times\left(\frac{T}{50}\right)^{1.226+u}} \quad (13)$$

Both parameters were significant at the 0.01 level, the explained variance R^2 was 0.94, the bias was -0.06, and the RMSE was 0.42 m. The standard deviation of the random plot factor (u) was 0.0000044. The index age was set to 50 years, meaning that the site index of birch plantations in Iceland corresponds to the dominant height at age 50. This reference age was selected after testing alternative index ages and was found to provide the best fit to the observed dominant height trajectories, particularly in terms of biological realism and stability of the site index curves across the available age range. When SI and stand age are known, the model can be used to calculate the dominant height for a certain site index:

$$\widehat{H} = \frac{15.364}{1-\left(1-\frac{15.364}{SI}\right)\times\left(\frac{50}{T}\right)^{1.226+u}} \quad (14)$$

Figure 4 shows that the model followed the patterns of the measured dominant heights of the sample plots used in this study.

According to the site index model, the maximum annual dominant height increment occurs at different stand ages depending on site productivity. On more productive sites (site indices 10 and 8), the peak dominant height increment is reached at younger ages, approximately between 10 and 15 years, whereas on intermediate sites (site index 6) it occurs later, around 15–20 years. On the least productive sites (site index 4), the peak increment is delayed further, occurring at approximately 20–25 years. These differences reflect the faster early height development on productive sites compared with poorer sites.

The age of maximum height increment is inferred from the modeled growth rates (i.e., the slope of the site index curves), rather than directly observed as a single point in Figure 4. At the reference age of 50 years, the modeled annual dominant height increment was low (<0.1 m yr^{-1}) for all site indices, indicating that height growth had largely slowed by this stage of stand development.

Height model

The selected Schumacher (1939) model for tree height was as follows:

$$\hat{h} = 1.3 + (-1.837 + (1.318 + u_1)Hdom) \times \exp\left[\frac{-\{-0.746 + (0.631 + u_2)Hdom\}}{d}\right] \quad (15)$$

The height-dbh relationship was well described by the selected mixed-effects model. The RMSE, when using the random plot effects in the prediction, was 0.46 m, and the bias was 0.013 m, indicating a very slight average overestimation of tree height by 1.3 cm. The standard deviations of u_1 and u_2 were 0.0988 and 0.1198, respectively, and the correlation between u_1 and u_2 was 0.675.

The model explained 91% of the variation in tree height (R^2 = 0.91). All estimated parameters were statistically significant at the 0.001 level. The residuals were approximately normally distributed and exhibited constant variance across the dbh range, with no major heteroscedasticity or pattern detected (Figure 5). A slight overestimation was observed for the tallest trees, which may reflect the limited representation of large trees in the dataset. Nonetheless, the model performs reliably across the full range of observed dbh and is suitable for practical applications in height prediction.

Diameter increment model

The selected dbh increment model was as follows:

$$\hat{i_d} - (1.944 + u_0) + 0.503\sqrt{d} - 0.430\left(\frac{d}{10}\right)^2 (-0.127 + u_1)\ln(G) + 0.798\ln(SI) + (-0.083 + u_2)\frac{BAL}{\sqrt{d+1}} \quad (16)$$

where is the future 5-year dbh increment (cm), d is dbh (cm), G is the stand basal area (m^2 ha^{-1}), SI is the site index (m), BAL is the basal area in trees larger than the subject tree (m^2 ha^{-1}) and u_0, u_1 and u_2 are random plot effects

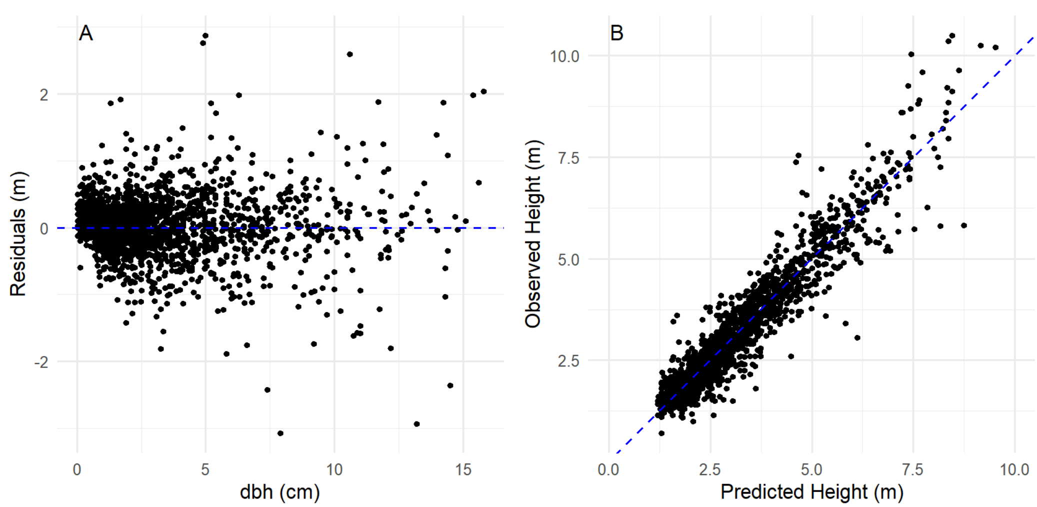

Figure 5. A) Residuals (observed-predicted) in predicting tree height plotted against diameter at breast height (dbh) and B) the correlation between predicted and measured height for planted downy birch in Iceland.

(Table 2). When using the random plot effects in the prediction, RMSE was 0.69 cm and the bias was -0.016 cm, indicating a very slight average underestimation of diameter growth. The model explained 46% of the variation in diameter growth (R^2 = 0.46). All fixed-effect parameters were statistically significant at the 0.05 level. The inclusion of random plot effects captured unobserved variability among plots. The residual plots (Figure 6A) revealed no strong trends or heteroscedasticity, indicating an adequate model fit.

Table 2. Standard deviations and correlations of the random plot effects of the dbh increment model for planted downy birch in Iceland (Model 16).

Parameter	Standard deviation	u_0	u_1	u_2
u_0	0.309			
u_1	0.194	-0.272		
u_2	0.139	0.323	0.258	

In Figure 6A, residuals are more densely distributed at smaller dbh values due to the larger number of small trees in the dataset. While large individual residuals occur primarily among small trees, the overall variance of residuals does not increase systematically with dbh, indicating no strong evidence of heteroscedasticity. Figure 6B shows the residuals plotted against BAL (basal area of larger trees). No systematic pattern was observed, suggesting that the BAL variable effectively accounted for competition. Residuals were generally symmetrically distributed around zero in both plots, with no evidence of strong bias.

Biomass and stem volume models

The selected model for aboveground biomass was:

$$\widehat{BM} = 0.123 \times (1 + d)^{2.235} \qquad (17)$$

The model for stem volume was as follows:

$$\hat{V} = 0.210 \times (d^2 \times h)^{0.784} \qquad (18)$$

Both models had a low bias and RMSE (Table 3). The explained variance (R^2) for the new aboveground biomass model was 0.90 and for

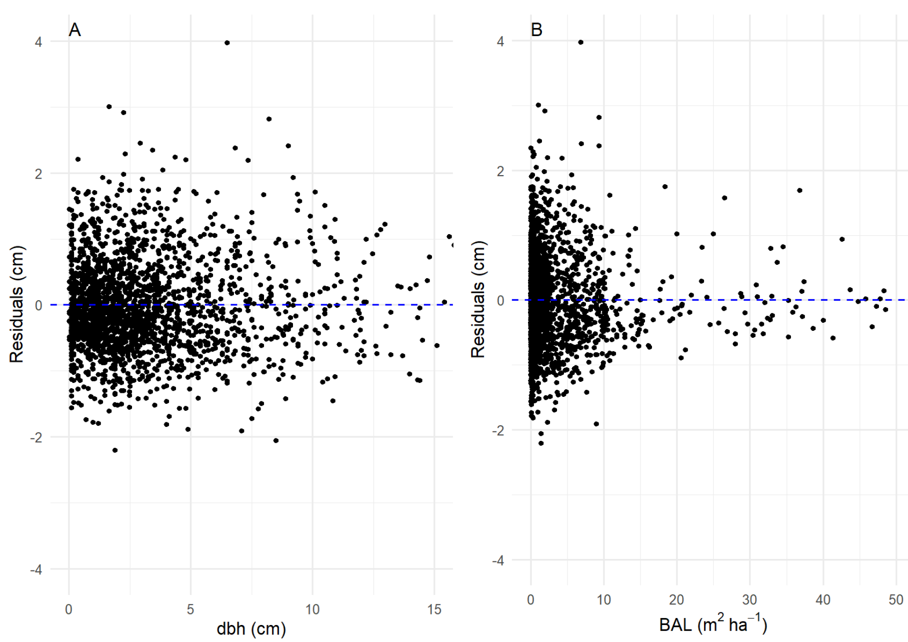

Figure 6. Residual plot of i_d (5-year dbh increments) prediction plotted against dbh (A), and model residuals plotted against BAL (basal area of trees larger than the subject tree, m^2 ha^{-1}) (B) for planted downy birch in Iceland.

the new stem volume model it was 0.91. Figures 7 and 8 demonstrate a strong linear relationship between predicted and observed values, with points closely following the 1:1 line, indicating good model calibration. For the aboveground biomass model, the bias was low (–0.14 kg) and RMSE was 7.42 kg, while the stem volume model had slightly higher bias (–1.30 dm³) and RMSE (9.01 dm³) (Table 3). Although both models slightly underestimated the largest observations, the overall fit remained good, with high R^2 values.

Table 3. Parameter estimates and fit statistics of the weighted models for individual tree aboveground biomass (17) and stem volume (18) for planted downy birch in Iceland. Bias, root mean squared error (RMSE) and explained variance (R^2). All estimated parameters are significant ($p < .001$).

Model	AIC	Bias	RMSE	R²
18	222.0	-0.14	7.42	0.90
19	259.3	-1.30	9.01	0.91

Simulation examples

The dbh increment model and the new biomass model were used to simulate stand development for five plots in the dataset, representing different site indices and stand basal areas in Icelandic birch plantations. Figure 9 presents the comparison between observed and predicted aboveground biomass accumulation over time across different SI classes. Simulations closely followed the observed biomass trajectories, particularly for plots with low and intermediate site indices (SI 4–9), demonstrating the model's ability to reproduce realistic growth predictions under typical growing conditions. For the highest productivity class (SI 10), the model overestimated biomass development. This is likely because of an overestimated site index or underrepresentation of high productivity stands in the dataset, which may have limited the model's capacity to fully capture growth under the most favorable site conditions.

Despite this, the models showed satisfactory performance across the observed range of site productivities and stages of stand development.

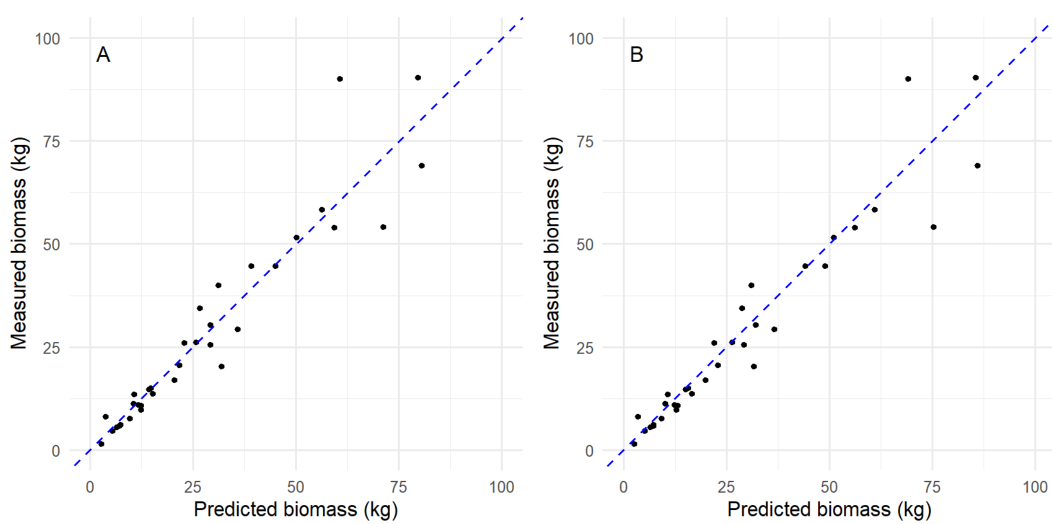

Figure 7. A) A scatter plot and 1:1 line for measured and predicted aboveground biomass (kg tree^{-1}) for the new model and B) the existing model by Snorrason and Einarsson (2006), where diameter at 0.5 m is used as the predictor, for planted downy birch in Iceland.

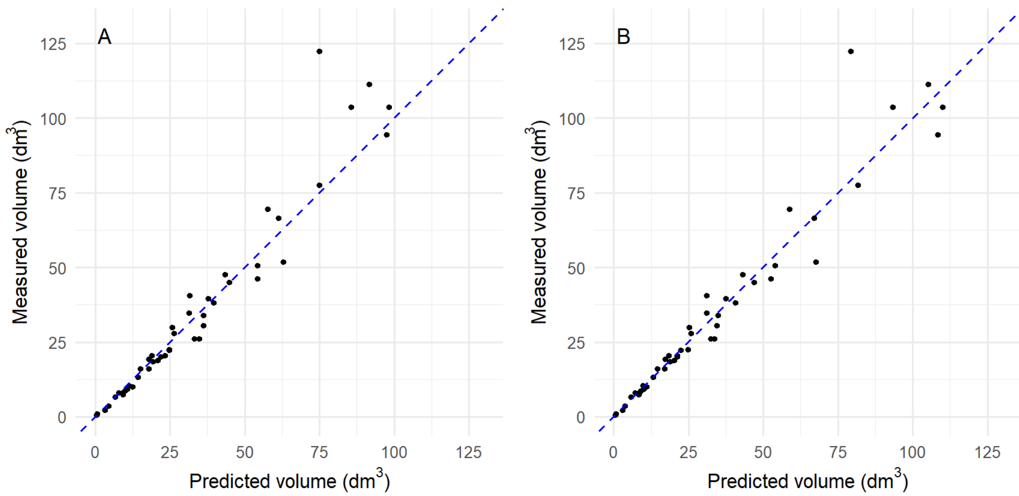

Figure 8. A) A scatter plot and the 1:1 line for measured and predicted stem volume (dm^3 tree^{-1}) for the new model and B) the existing model by Snorrason and Einarsson (2006), where diameter at 0.5 m was used as a predictor for planted downy birch in Iceland.

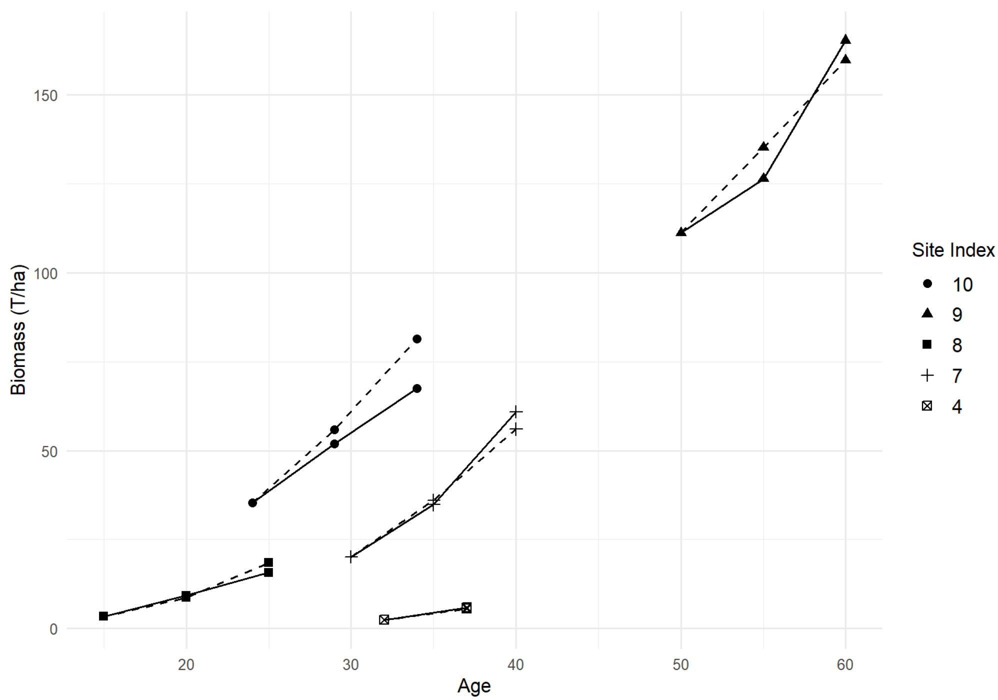

Figure 9. Simulated and observed aboveground biomass accumulation (t ha^{-1}) over time across different site index (SI) classes in Icelandic downy birch plantations. Solid lines with markers represent observed biomass values derived from measured tree diameters, while dashed lines show model predictions. Predicted biomass trajectories are based on tree diameters simulated using the dbh increment model, which were subsequently converted to aboveground biomass using the dbh-based allometric biomass model. Marker shapes correspond to different SI levels.

DISCUSSION

This study presents a set of models that enable, for the first time, a tree-level, distance-independent simulation of the development of pure even-aged downy birch stands in Iceland. In addition, new tree-biomass and volume models were developed that use dbh as a predictor instead of diameter measured at 0.5 m above ground, which was used in earlier models. The models are based on data from permanent sample plots, established by the Icelandic National Forest Inventory and measured 2–5 times at 5-year intervals, covering a range of the realized different climate and soil conditions across Iceland. They should therefore give robust country-wide predictions.

Because the sampling design was not specifically developed for growth and yield modelling, some limitations exist. In particular, stem diameter was not always repeatedly measured at a permanently marked height or in a consistent direction at different measurement occasions. Such practices are known to increase measurement error in estimated diameter increment when compared with protocols based on fixed measurement points, multiple diameter directions, or circumference measurements (Van Laar & Akça 2007, West 2009, Avery & Burkhart 2015).

The diameter conversion model between measurements taken at 0.5 m above ground and diameter at breast height showed a high degree of explained variance and very low bias, indicating a stable and predictable relationship between the two measurement heights. This supports the practical use of diameter measurements at 0.5 m in Icelandic birch inventories, where stem irregularities, multi-stemming, or snow damage may complicate measurements at breast height. Similar linear or near-linear relationships between diameters measured at different stem heights have been reported in other studies, suggesting that such conversion models can provide reliable compatibility between historical datasets and standard dbh-based models when direct dbh measurements are unavailable. The relatively small residual variation observed in this study further indicates that the conversion is unlikely to introduce substantial additional error into growth, biomass, or volume predictions (e.g., West 2009, Picard et al. 2012, Avery & Burkhart 2015).

The selection of the SI model was guided by biological consistency and practical applicability, with particular emphasis on realistic asymptotic behavior and reasonable extrapolation beyond the observed age range. The model captured the general growth trends of dominant height in Icelandic downy birch stands (Figure 4). The estimated asymptote of 15.36 m should not be interpreted as a commonly observed maximum height under current Icelandic conditions, as native downy birch trees taller than 15 m are rare. Instead, the asymptote represents the upper limit of the growth trajectory implied by the model under favorable site conditions.

In a broader Nordic context, dominant heights of downy birch exceeding 15 m are reported on productive sites in Finland, Sweden, and Norway (Braastad 1967, Johansson 1999, 2003, Bollandsås et al. 2008, Hynynen et al. 2010), indicating that the estimated asymptote is biologically reasonable for the species. The lower realized dominant heights observed in Iceland likely reflect a combination of environmental constraints, such as a harsh climate and short growing season, as well as genetic factors. In particular, Icelandic downy birch populations are frequently hybridized with dwarf birch (*Betula nana*), a shrub-form species (Anamthawat-Jónsson et al. 2021), which has been shown to contribute to reduced height growth and inferior stem form (Snorrason et al. 2016).

However, selected birch provenances used in plantation forestry, such as Bæjarstaðarbirki and Bæjarstaðarúrval, are characterized by superior stem quality and greater height growth compared with the average native birch population (Eysteinsson et al. 2023). Under favorable site conditions, these provenances may reach dominant heights approaching 15 m, suggesting that the estimated asymptote represents a plausible upper limit for improved birch material currently used in Icelandic forestry, even if such heights remain uncommon in unmanaged native stands.

The predictions of the dominant height model for stands younger than 10 years should be interpreted with caution. Height development in early stand development phases is influenced not only by site productivity but also by microsite variability, competition, and regeneration conditions (Borders et al. 1984, Barrio Anta & Diéguez-Aranda 2005). Despite this, the model provides a robust framework for estimating site productivity across a wide range of conditions and stand ages typical of Icelandic birch forests.

The height–dbh relationship plays a fundamental role in forest growth and yield modelling, particularly when tree heights are not measured but are required for applications such as volume and biomass estimation (e.g., Curtis 1967, Van Laar & Akça 2007). The height model developed in this study showed a high explained variance (R^2 = 0.91), a low RMSE of 0.46 m, and a very small bias of 1.3 cm, indicating that the model captures the overall height–dbh relationship with minimal systematic error. Similar levels of accuracy have been reported for height–dbh models for birch and other broadleaved species in the Nordic region (Johansson 1999), suggesting that the performance of the present model is well within the range expected for operational forest modelling. The results therefore support the use of the developed height model for predicting missing tree heights in Icelandic birch stands.

A slight overestimation of tree height was observed for the largest trees, which likely reflects a combination of model-form limitations and the underrepresentation of large-diameter trees in the available data. Such behaviour in the upper tail of the diameter distribution is common in height–dbh modelling when asymptotic or sigmoidal functions are fitted primarily to small and medium-sized trees. Model performance for large trees could be improved by extending the calibration dataset to include a greater number of dominant and large-diameter individuals, either through targeted sampling or by combining national inventory data with data from experimental plots. In addition, alternative model formulations that allow greater flexibility in the upper diameter range, such as mixed-effects height models, generalized additive models, or segmented and taper-based approaches, may better accommodate variation among large trees and reduce systematic bias. Previous studies have shown that such approaches can improve height prediction for large trees without compromising overall model stability (e.g., Sharma & Parton 2007, Temesgen et al. 2008, Mehtätalo & Lappi 2020).

From an application perspective, the height–dbh model is suitable for use in forest inventory and individual-tree growth simulation. Height models are commonly used to predict missing tree heights in inventories where only a subset of trees is measured for height, and to support the estimation of tree volume, biomass, and site index (e.g., Van Laar & Akça 2007, Temesgen et al. 2008). The inclusion of plot-level random effects further enhances the applicability of the model by allowing local calibration when a small number of height measurements are available from a new plot. This approach has been shown to substantially improve height prediction accuracy and volume estimation in operational forestry, particularly in national forest inventories and management planning, where the height measurement of all trees is impractical (de-Miguel et al. 2012, Mehtätalo et al. 2015).

The mixed-effects model for predicting dbh increment showed robust performance, with a residual standard deviation of 0.7 cm, which is comparable to values reported for individual-tree diameter increment models for birch in Scandinavia and the UK (e.g., Hynynen et al. 2002, Broad & Lynch 2006). For silver birch (*Betula pendula*), residual standard deviations in the range of approximately 0.6–1.0 cm have commonly been reported, depending on stand structure and measurement interval (Soares & Tomé 2002, Broad & Lynch 2006). The inclusion of site productivity (SI), stand basal area, and the basal area of larger trees allowed the model to explicitly describe the effects of site quality, overall stand density, and asymmetric competition on individual-tree diameter growth, which are well-established drivers of tree growth variation (von Gadow & Hui 1999, Weiskittel et al. 2011). The inclusion of plot-level random effects further improved model accuracy by capturing unobserved between-plot differences in growth conditions, such as local climate, soil properties, and management history, an approach that has been widely applied and shown to improve diameter increment modelling in boreal and temperate forests (Pinheiro & Bates 2000, Calama & Montero 2004, Weiskittel et al. 2011). Together, these results suggest that diameter growth in planted Icelandic birch forests is primarily constrained by site productivity at broader spatial scales, while competition-related effects largely determine growth differentiation among trees within stands.

The residual analysis (Figure 6) indicated that the dbh increment model performed consistently across the observed range of tree sizes and competitive conditions, with no strong unexplained patterns with respect to BAL, suggesting that competition-related growth suppression was adequately represented within the data range. However, no tree mortality was observed in the dataset, and the model therefore describes the diameter increment of surviving trees only. As a result, mortality processes could not be parameterized and are not implicitly captured through changes in diameter increment or site index with age.

The dataset primarily represents young to moderately stocked, planted birch stands, with basal area values below approximately 50 m^2 ha^{-1} (Table 1). Although this upper value is relatively high, density-dependent mortality and self-thinning in even-aged stands typically occur at substantially lower basal areas, depending on species, site productivity, and stand structure (Reineke 1933, Pretzsch 2009). As no tree mortality was observed in the study plots during the measurement intervals, mortality processes could not be parameterized. Consequently, while the present model can be reliably applied to predict diameter increment across stand structures typical of planted birch forests in Iceland, it should be used in combination with an explicit mortality component when applied in long-term simulations of highly stocked or unmanaged stands, to avoid unrealistically high stand densities and overestimation of stand development.

The updated total biomass and stem volume models showed satisfactory predictive performance, as indicated by low bias and RMSE values and high explained variance (R^2). The models were developed using partly the same underlying dataset as Snorrason and Einarson (2006), allowing a comparison of model performance. Compared with the earlier study, the explained variance was slightly lower for both total aboveground biomass (0.90 vs. 0.96) and total stem volume (0.91 vs. 0.99). An important difference between the two modelling approaches is that tree height was not statistically significant in the updated aboveground biomass model and was therefore excluded, whereas it was a significant predictor in the earlier models, which were based on diameter measured at 0.5 m above ground. In contrast, tree height remained a significant variable in the updated total stem volume model. The exclusion of height from the biomass model likely contributes to the lower R^2 compared with the previous study, as including height can reduce residual variance by accounting for vertical growth differences among trees. However, omitting height simplifies the model and increases its operational applicability in forest inventories, where height measurements are time-consuming and often subject to greater measurement error than dbh.

The modelling choices adopted here are consistent with common practice in Nordic and UK forestry. In Finland, Sweden, Norway, and the UK, stem volume models are typically based on both dbh and height, reflecting the strong geometric relationship between these variables and merchantable volume, whereas aboveground biomass models often rely primarily on dbh alone, particularly for young and medium-sized trees (e.g., Marklund 1988, Johansson 1999, Zianis et al. 2005, Repola 2009). Several studies have shown that dbh is the dominant predictor of tree biomass, as it integrates the effects of radial growth and correlates strongly with wood volume and density, while height adds relatively limited additional explanatory power for biomass estimation (Návar 2009, Repola 2009, Picard et al. 2012).

Visual inspection of the relationship between predicted and measured stem volume indicated a tendency for underestimation at the upper end of the distribution, particularly for larger trees (Figure 8A). This suggests that model performance for large trees could be improved by incorporating additional observations from older or larger-diameter trees or by exploring alternative nonlinear model forms better suited for the upper tail of the distribution. Nevertheless, the updated models represent a practical improvement by using dbh as a predictor, aligning with international modelling standards and enhancing their applicability for forest inventory and biomass estimation in Icelandic downy birch forests.

In the simulation example, the biomass model reproduced the observed trends in aboveground biomass accumulation across site index classes, demonstrating robust performance when applied in combination with the diameter increment model. It should be noted that the site index does not enter the biomass model directly, as aboveground biomass was estimated using a dbh-based allometric equation. Consequently, the separation of biomass trajectories among SI classes arises indirectly through differences in simulated diameter development driven by site productivity and stand dynamics, rather than from an explicit effect of site index in the biomass model itself.

A slight overestimation of biomass was observed for the highest site index class (SI 10), which is consistent with the earlier discussion of increased uncertainty at the upper end of site productivity due to the limited number of highly productive plots available for model calibration. Similar behaviour has been reported in other empirical growth and biomass models, where predictions are less reliable under sparsely represented stand conditions (Weiskittel et al. 2011, Burkhart & Tomé 2012). Nevertheless, the close agreement between simulated and observed biomass for low and intermediate SI classes suggests that the combined modelling framework performs well under typical Icelandic growing conditions, where productivity is generally moderate and stand development follows more consistent trajectories.

In summary, this study developed a coherent set of models for site index, height–dbh relationships, dbh increment, stem volume, and aboveground biomass that together enable tree-level, distance-independent simulation of growth and yield in planted Icelandic downy birch stands. The models are biologically consistent and were evaluated using both statistical performance measures and graphical diagnostics, demonstrating reliable behavior within the range of site conditions and stand structures represented in the data.

Some limitations were identified, including increased uncertainty for the highest site index classes due to limited data from highly productive sites and the exclusion of tree height from the final biomass model. However, these limitations are well defined and do not compromise model performance under typical Icelandic growing conditions, where site productivity is generally moderate and height measurements are often unavailable. The resulting modelling framework therefore provides practical and robust tools for forest inventory, growth simulation, and management planning using native birch.

Future work should focus on expanding the empirical basis of the models by incorporating additional data from highly productive stands and older age classes, as well as evaluating long-term model behavior under changing environmental conditions, to further strengthen their applicability in Icelandic forestry. In addition, models for self-thinning or trees survival probability need to be developed.

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