

Individual-tree growth models for lodgepole pine (*Pinus contorta*) in Iceland

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ABSTRACT

Lodgepole pine (*Pinus contorta* subsp. *contorta*) is one of the most important tree species planted in Iceland. The current plantation area is 7100 ha, the first plantations being 80 years old. This study presents models for simulating the development of Icelandic lodgepole pine plantations on an individual-tree basis. The model set consists of a site index model, tree height model, and diameter increment model. Data were collected in 35 permanent sample plots with measurement intervals ranging from 3 to 14 years. The total number of diameter increment observations was 3664. Regression analysis was used in site index and height modelling, and both regression analysis and optimization were tested in diameter increment modelling. An optimization-based model was evaluated to be the most suitable for growth simulations. The use of the developed model set was demonstrated in management optimizations, where the rotation length and cutting schedule were optimized for two young sample plots, one representing poor sites and the other good sites. The optimizations showed that planting is not profitable on poor sites if the discount rate is 4% or higher. The mean annual stem wood harvest was 2.7–4.1 m³ha⁻¹a⁻¹ on the poor site and 8.3–11.7 m³ha⁻¹a⁻¹ on the good site.

Keywords: Site index model, height model, diameter increment model, optimization-based modelling

YFIRLIT

Jöfnur sem lýsa vexti stafafuru (Pinus contorta) á Íslandi

Stafafura (*Pinus contorta* subsp. *contorta*) er ein af þeim trjategundum sem mikið hefur verið gróðursett af á Íslandi síðastliðin 80 ár og þekur tegundin í dag um 7100 hektara. Í þessari grein voru þekktar erlendar vaxtarjöfnur sem áætla vöxt stakra trjáa við mismunandi vaxtarskilyrði aðlagðar að íslenskum aðstæðum. Jöfnurnar samanstanda af nokkrum jöfnum sem lýsa gróskustigi, hæðarvexti stakra trjáa og þvermálsvexti. Gögnin sem notuð voru við aðlögun jafnanna var safnað af 35 föstum mæliflötum, þar sem sömu trén hafa verið endurmæld á 3 til 14 ára fresti og hafa mælingar í elstu mæliflötunum verið gerðar frá árinu 1966. Samanlagður fjöldi þvermálsvaxtarmælinga var 3664. Aðhvarfsgreining var notuð við aðlögun á jöfnum fyrir gróskustig og hæðarvöxt stakra trjáa og bæði aðhvarfsgreining og bestunarnálgun (optimization approach) voru prufuð við aðlögun þvermálsvaxtarjöfnunnar. Bestunar nálgun þótti lýsa best áætluðum þvermálsvexti. Til að sýna notkun á jöfnunum sem voru aðlagðar, voru þær látnar áætla vöxt skóga fram að endurnýjun skógarins og voru mælingar frá tveimur ungum skógum notaðar til þess þar sem annar var á gróskumiklu landi og hinn á gróskulitlu landi. Niðurstöðurnar sýndu að á gróskuminna landinu var viðarframleiðsla ekki hagkvæm þegar ávöxtunarkrafan var 4% eða hærrí. Meðal árlegur viðarvöxtur var á bilinu 2,7-4,1 m³ha⁻¹a⁻¹ í gróskuminni skóginum en á bilinu 8,3-11,7 m³ha⁻¹ í gróskumeiri skóginum.

INTRODUCTION

In recent years, a forest planning system for sustainable forest management has been under development in Iceland. The system is called IceForest, and it is a modification of the Finnish Monsu system (Pukkala 2004, Heinonen et al. 2018, Diaz-Yáñez et al. 2020). The main idea of forest management planning is to discover how the forests should be managed in such a way that the benefit or utility of the forest owner is maximized (Pukkala 2002). This is done by finding a combination of the treatment schedules of stands that best meet the management objectives set for the forests (Kangas & Kangas 2001). The main steps and tools in planning are inventory, data management, simulation of stand development under alternative management schedules, and finding the optimal combination of the management schedules for the stands or other calculation units (Pukkala 2002). Normally, the calculation units are tree stands but, especially in strategic planning, they may also be inventory plots or strata (e.g. Heinonen et al. 2018).

An important part of the system is reliable growth models for yield prediction and for the simulation of stand development. A growth model is an abstraction of the natural dynamics of a forest stand and may encompass tree growth, mortality, regeneration, and other components of stand dynamics. Generally, it refers to a system of equations that can predict the growth and yield of a forest stand under a wide variety of conditions (Vanclay 1994). Growth models provide forest researchers and managers with an efficient way to make resource forecasts, but their more important role may be their ability to explore management options and silvicultural alternatives (Vanclay 1994).

According to Munro (1974), forest growth and yield models can be divided into three broad categories: stand-level models, individual-tree models, and diameter distribution models. Stand-level models are developed using stand-level information (Curtis et al. 1981, Vanclay 1994), whereas individual-tree models consider individual tree growth (Clutter et al. 1983, Palahí et al. 2003). Diameter distribution models use statistical probability functions to

characterize the stand structure (Bailey & Dell 1973, Newton et al. 2005). Tree-level models are further classified as distance-dependent or distance-independent models.

Lodgepole pine (*Pinus contorta* subsp. *contorta*) is one of the most planted tree species in Icelandic forestry, covering an area of 7100 ha or 15% of the cultivated forest in Iceland (data from the Icelandic National Forest Inventory, NFI). The capability of lodgepole pine to grow on a wide range of soils and elevations, as well as its growth potential, are reasons why lodgepole pine is planted for timber production, carbon sequestration, and soil reclamation. At present, no growth models exist for lodgepole pine in Iceland. Given the importance of lodgepole pine, there is a need for a reliable system of growth models that would allow managers to predict harvests and future stand development for different thinning treatment schedules, thus providing valuable support for silvicultural decision-making. The objective of this study was to develop a tree-level, distance-independent growth model for simulating the growth of lodgepole pine plantations in Iceland.

MATERIALS AND METHODS

Site conditions

The data used in this study came from two projects: permanent sample plots (PSP) established for growth measurements and permanent National Forest Inventory (NFI) plots. The NFI plots were placed in random locations, whereas the PSP plots were located in the inner parts of subjectively-selected stands to minimize the edge effect of the forest border. The PSP data consisted of measurements in 9 stands at 3 locations (Figure 1). The NFI data were collected from 26 permanent sample plots in 17 locations (Figure 1). All plots were in planted, even-aged lodgepole pine stands (*Pinus contorta* subsp. *contorta*) established by the Icelandic Forest Service between 1966 and 2009 in young plantations and included a total of 110 measurement intervals. Only a few of the older sample plots had been thinned and most of the plots were unthinned. The thinnings were

light from below. Suppressed and low-quality trees were removed.

The data covered a wide range of different site types and growth conditions from thinned to a few unthinned stands. All the locations have an oceanic climate with an annual precipitation (1964–1990) of 700–1200 mm and a mean annual temperature of 3.2–4.5°C (Vedurstofa Islands 2017). For the same period, the mean maximum daytime temperature during June–August was 12.9–13.6 °C (Vedurstofa Islands 2017). The range in plot elevation was between 10 and 140 m a.s.l.

The sample plots were either circular or square-shaped. The size of the PSP plots varied between 0.012 and 0.05 ha and were in locations where forestry had been practiced for 50 years or longer. The size of the NFI plots varied between 0.005 and 0.02 ha, and the plots were mainly located in areas where tree planting and other forest activities were more recent. One reason for the small plot size was the limited

Table 1. Mean, standard deviation (SD) and range of the main characteristics in the empirical data of the study material. N: number of observations; dbh: diameter at breast height; G: stand basal area; Age: stand age; Hdom: dominant height.

Variable	N	Mean	SD	Maximum	Minimum
dbh (cm)	4006	10.27	6.64	40.5	0.0
Height (m)	3957	6.1	4.02	21.9	0.1
G (m ² ha ⁻¹)	119	19.31	19.49	83.9	0.0
Age (years)	119	28.8	14.6	79.0	7.0
Hdom (m)	119	6.88	4.48	21.27	0.62
Growth periods	119	5.2	1.5	14.0	3.0
Stems per hectare	119	1858	1332	6333	290

size of Icelandic tree plantations. To minimize edge effects, small sample plots are common in Icelandic tree growth data.

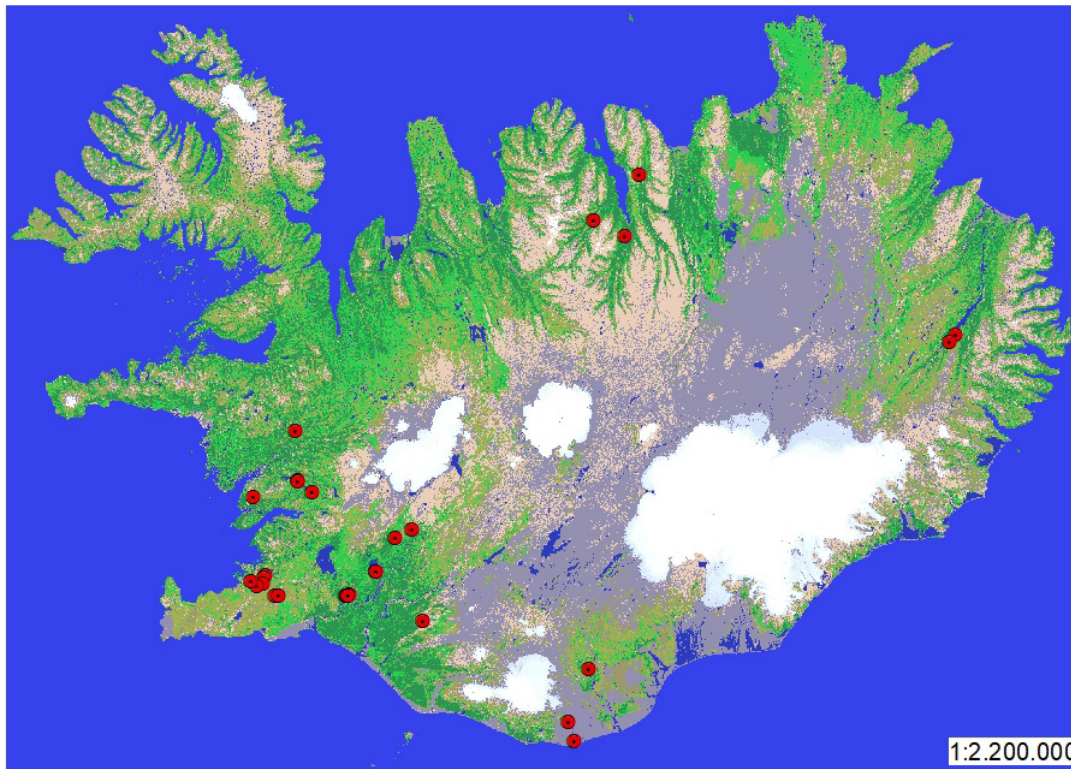


Figure 1. Geographical locations of the study sites in Iceland.

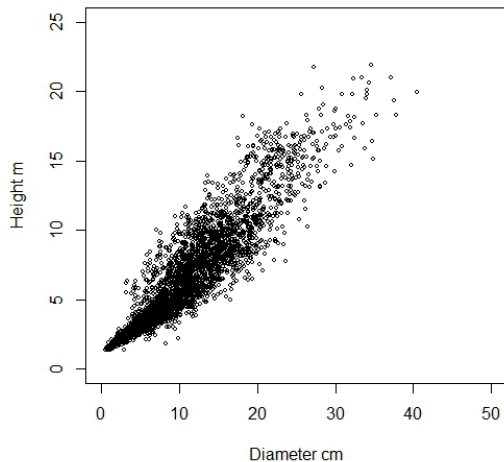


Figure 2. Observed heights and diameters at breast height in the lodgepole pine plots used in the study.

The measurement interval ranged from 3 to 14 years with an average of 5.2 years. On every measurement occasion on the PSP plots, two measurements of diameter at breast height (DBH, at 1.3 m) at right angles were done using callipers, and the arithmetic mean of the two measurements was calculated. The total tree height was measured on all standing trees of the PSP plots using a height measuring pole or Vertex Laser VL5 instrument. On the NFI plots, DBH was measured in one direction. Double measurement was only used if the tree stem was perceptibly not circular. On some NFI plots the total tree height was measured only on sample trees. The tree selection for height measurements was based on DBH, and the aim was to get heights from different DBH classes. Height was measured with a measuring pole for trees shorter than 4 m and with a Laser Tech distance and height measurement instrument for taller trees.

The use of the developed model set was demonstrated in management optimizations where the rotation length and cutting schedule were optimized for two young sample plots, one representing poor sites and the other good sites. In the optimization example, it was assumed that the management schedule included three thinning treatments during the rotation.

The stand establishment costs were as follows: site preparation 45.382 IKR/ha; seedlings 250.000 IKR/ha; planting 108.750 IKR/ha (1 € is about 150 IKR). The timber assortments were sawlog with minimum piece length of 3 m, minimum top diameter of 14 cm, and stumpage price of 20.000 IKR/m³; chip/pulp log (3 m, 6 cm, 10.000 IKR/m³; and firewood (3 m, 3 cm, 5.000 IKR/m³).

Site index modelling

The model set developed in this study consisted of a site index model, tree height model, and diameter increment model. Because there was very little mortality in the dataset, even at a very high basal area, no attempt was made to model tree survival. The site index model was used to calculate the site index of every sample plot. It was also used to predict dominant height development. Dominant height was used as a predictor in the tree height model, which facilitated simulations in which stand dominant height and individual tree heights were logically related to each other (DeMiguel et al. 2013, Mehtätalo et al. 2015).

Only the first and the last age and dominant height measurements of each plot were used to fit the site index model. Therefore, there were no correlated observations, and no need to use mixed-effects modelling. Several functions commonly used in the algebraic difference approach (ADA) were tested (Palahí et al. 2004). The tested functions were: Korf and Lundmark (Korf 1939), three different versions of the Schumacher model (see Palahí et al. 2004), three versions of the Chapman-Richards model (Palahí et al. 2004), the model of McDill and Amateis (1992), and the model of Hossfeld (Peschel 1938). The model that minimized the RMSE and the Akaike Information Criterion (AIC) was the following McDill-Amateis function:

$$H_2 = \frac{b_0}{1 - (1 - b_0/H_1)} \times \left(\frac{T_1}{T_2}\right)^{b_1} + \varepsilon \quad (1)$$

where T_1 and H_1 are, respectively, the stand age and dominant height at the first measurement occasion and T_2 and H_2 are the same variables

at the second measurement occasion. Dominant height was defined to be the mean height of 100 largest trees (in DBH) per hectare and was calculated as the mean height of the largest trees of the plot, the number of dominants depending on plot area. Site index was defined to be dominant height at 80 years. The index age of 80 years has been used in Iceland previously for larch (Heiðarsson & Pukkala 2012) and Sitka spruce plantations (Heiðarsson et al. 2022).

The site index model was used to calculate the site index for every plot of the dataset. Because the plots were measured several times, the site index was calculated at every measurement occasion, and the mean site index was used as a predictor in growth modelling.

Height modelling

The two-parameter versions of the Näslund (1936), Schumacher (1939), and Curtis (1967) functions analysed in Mehtätalo et al. (2015) were tested as candidate models to describe the relationship between tree diameter and height. The parameters of the models were expressed as a function of dominant height, which allowed the height curve to change along stand development. As the first step, fixed-effect models were fitted. These fittings showed that the best function according to RMSE and AIC was the following Schumacher model:

$$h = 1.3 + (a_0 + a_1 H_{dom}) \times \exp\left[-\frac{b_0 + b_1 H_{dom}}{d}\right] + \varepsilon \quad (2)$$

where h is the tree height and d is DBH. This model was further developed by adding random plot factors to the parameters. The best combination of random effects was found by trial and error, by testing all possible combinations of random effects. The best mixed-effects model included random plot factors for parameters a_1 , b_0 and b_1 .

Diameter increment modelling

Two different methods were used to fit models for diameter increment: regression analysis and the optimization-based approach suggested by Pukkala et al. (2011), which was used earlier in Iceland by Heiðarsson et al. (2022) for

Sitka spruce. Since the interval between plot measurements varied from 3 to 14 years, the measured period of growth was divided by the length of the measurement interval, to obtain the mean annual diameter increment of the period.

As the first step, regression analysis and fixed-effects modelling were used to search for the best transformations and combinations of predictors. The model had to include at least one predictor that described tree size (different transformations of DBH were used), at least one variable that described competition (stand basal area and basal area in larger trees were tested), and a variable that described site productivity (site index). The following model turned out to be the most satisfactory:

$$i_d = \exp\left[a_0 + a_1 \ln(d) + a_2 \left(\frac{d}{10}\right)^2 + a_3 \ln(G) + a_4 \ln(SI) + a_5 \frac{BAL}{\sqrt{(d+1)}} + a_6 \ln(t)\right] + \varepsilon \quad (3)$$

where i_d is the mean annual diameter increment of the measurement period (cm), d is DBH (cm), G is the stand basal area (m^2ha^{-1}), SI is the site index (m), BAL is the basal area of trees larger than the subject tree (m^2ha^{-1}) and t is tree age (years). Because the plots were measured in even-aged plantations, the tree age was equal to the number of years since planting.

Then, the same model was fitted as a mixed-effects model, by adding a random plot factor to parameter a_0 . In addition, the model was fitted using the optimization-based approach of Pukkala et al. (2010). In this approach, the tree diameters of the first measurement were used to start a simulation in which the diameter increment model was used to simulate tree growth from the first measurement to the second measurement, using a one-year time step. All measurement intervals of all plots were simulated in this way.

The diameter distributions of trees at the second measurement that were obtained from simulations were compared to the measured diameter distributions. This simulator was linked with an optimization algorithm that gradually adjusted the parameters of the diameter increment model. Differences between simulated and measured diameter distributions

were minimised via loss functions. The algorithm of Nelder and Mead (1965) was used in parameter optimization. The optimization was repeated with three alternative loss functions:

Loss function 1 was as follows:

$$\min z(\theta) = \sum_{k=1}^K \left[\sum_{j=1}^{J_k} w_{jk} \sum_{i=1}^{I_j} |g_{ijk}^m - g_{ijk}^s(\theta)|^{1.5} + 0.001 |n_{ijk}^m - n_{ijk}^s(\theta)|^{1.5} \right] \quad (4)$$

where Θ is the set of coefficients (a_0, \dots, a_6 of Equation 3) estimated as $\arg \min z(\Theta)$, K is the number of plots, J_k is the number of measurement intervals of plot k , I_j is the number of 3-cm diameter classes in measurement interval j of plot k , g_{ijk}^m and $g_{ijk}^s(\Theta)$ are, respectively, measured and simulated cumulative basal area (m^2ha^{-1}) of diameter class i at the end of measurement interval j of plot k , and n_{ijk}^m and $n_{ijk}^s(\Theta)$ are, respectively, the measured and simulated cumulative number of trees per hectare in diameter class i at the end of measurement interval j of plot k (see, e.g., de-Miguel et al. 2014 for details). Symbol w_{jk} is the weight of measurement interval j of plot k . The number of trees in plot k at the beginning of period j was used as the weight.

Loss function 2 was otherwise similar except that it used the measured and simulated frequencies of 3-cm diameter classes instead of cumulative frequencies.

Loss function 3 was simpler; it minimised the differences between simulated and measured stand basal areas at the end of the measurement interval:

$$\min z(\theta) = \sum_{k=1}^K \left[\sum_{j=1}^{J_k} w_{jk} |G_{jk}^m - G_{jk}^s(\theta)|^{1.5} \right] \quad (5)$$

In this loss function, G_{jk} is the total basal area of plot k and the end of measurement period j .

The analyses produced five different versions of the diameter increment model: a fixed-effects regression model, mixed-effects regression model, and three versions of the optimization-based model. These models were compared by calculating the RMSE and bias for the (back-transformed) annual diameter increment (cm

per year). The fixed part of the mixed-effects model was used in this analysis.

This model evaluation has limitations. The purpose of the models was to predict the diameter increment of the next year but, in model evaluation, the predictions were compared to the average annual growth of a period ranging from 3 to 14 years. Therefore, the models were tested also via simulation. The development of all plots was simulated from the beginning to the end of the measurement period, using each of the five models. Then, the simulation results were used to calculate the periodical increment of the stand basal area (m^2ha^{-1}). These increments were compared to the measured periodical increment of the stand basal area.

RESULTS

Site index model

The fitted parameters of the McDill-Amateis site index model (Equation 1) were: $b_0 = 38.7615$ and $b_1 = 1.5586$. The RMSE of the model was 0.97 m. The model is used as follows to calculate site index from stand age (T) and measured dominant height (H_{dom}):

$$\widehat{SI} = \frac{38.7615}{1 - (1 - 38.7615/H_{\text{dom}})} \times \left(\frac{T}{80}\right)^{1.5586} \quad (6)$$

When site index and stand age are known, dominant height is calculated from

$$\widehat{H}_{\text{dom}} = \frac{38.7615}{1 - (1 - 38.7615/SI)} \times \left(\frac{80}{T}\right)^{1.5586} \quad (7)$$

Figure 3 shows that the model follows the patterns of the measured dominant heights on the sample plots used in this study.

Height model

The fitted Schumacher model for tree height was as follows:

$$\hat{h} = 1.3 + (-1.097 + u_{1k} + 1.216H_{\text{dom}}) \times \exp \left[-\frac{2.033 + u_{2k} + (0.343 + u_{3k})H_{\text{dom}}}{d} \right] \quad (8)$$

where h is the tree height, d is DBH, and u_{1k} , u_{2k} and u_{3k} are random factors for plot k (Table 2).

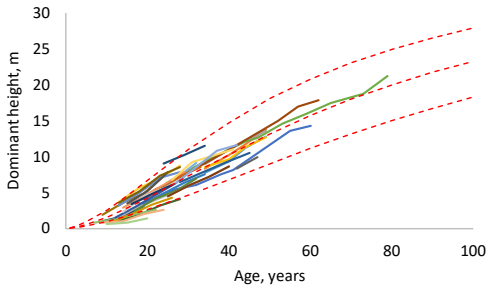


Figure 3. Dominant height curves for site indices 15, 20 and 25 m (site index = dominant height at 80 years) and the measured age and dominant height sequences of the study plots. Only the first and last measurements of the plots were used to fit the site index model.

Table 2. Standard deviations and correlations of the random plot effects of the height model (Equation 8).

Standard deviations		Correlations		
u_{1k}	0.090	u_{1k}	u_{1k}	u_{2k}
u_{2k}	1.103	u_{2k}	0.116	
u_{3k}	0.229	u_{3k}	0.114	-0.960
Residual	0.735			

The standard deviation of the residual of the full model (random effects used in predictions) was 0.735 m (Table 2). When predictions were calculated with the fixed part of the mixed-effects model (assuming that the random effects are zero), the standard deviation was 0.828 m. The bias of the fixed part of the mixed-effects model was -0.159 m, i.e., the model underestimated tree height on average by 15.9 cm. As the parameters of the two-parameter

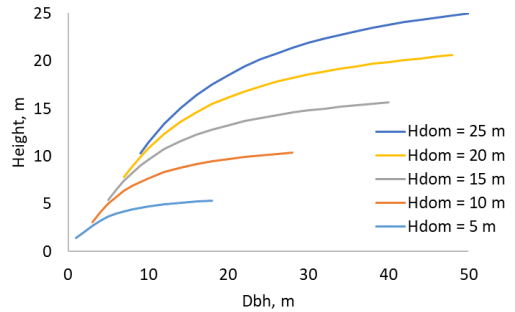


Figure 4. Relationship between tree diameter and height at different dominant heights according to the height model (Equation 8).

Schumacher model (Mehtätalo et al. 2015) were expressed as a function of dominant height, the relationship between DBH and height changed with the increase in dominant height (Fig. 4).

Diameter increment model

The parameters of the diameter increment model (Equation 3) were estimated five alternative ways, three of which represented the optimization-based approach (Pukkala et al. 2011). The fitting statistics (RMSE and bias) calculated for mean annual diameter increments of the trees suggested that the fixed-effects regression model was the best when the mixed-effect model was used without random plot factors (Table 3).

However, the statistics of Table 3 were based on the average annual increment of periods of 3–14 years, although the purpose of the model is to predict the diameter increment of the next year. Hence, the results of Table 3 do not prove that the fixed-effects regression model is the best for simulations. Therefore, the model

Table 3. Fitting statistics of alternative diameter increment models in tree-level analysis.

	Regression models		Optimization-based models		
	Fixed	Mixed	Loss 1	Loss 2	Loss 3
RMSE, cm/year	0.196776	0.206978	0.214328	0.20663	0.223371
RMSE, %	40.3	42.4	43.9	42.4	45.8
Bias, cm/year	-0.00023	0.012337	0.041784	-0.00594	0.034891
Bias, %	-0.05	2.53	8.56	-1.22	7.15

Table 4. Fitting statistics of alternative diameter increment models in plot-level analysis.

	Regression models		Optimization-based models		
	Fixed	Mixed	Loss 1	Loss 2	Loss 3
RMSE, m ² ha ⁻¹	1.822829	2.121951	1.707001	2.041182	1.67652
RMSE, %	25.9	30.19	24.29	29.0	23.8
Bias, m ² ha ⁻¹	-0.52721	-0.17837	0.072692	-0.82135	0.144135
Bias, %	-7.5	-2.55	1.05	-11.7	2.0

Table 5. Coefficients of the diameter increment model (Equation 3) obtained with two different methods.

Parameter, predictor	Fixed-effects regression model	Optimization-based model, loss function 1
a0, intercept	0.19482	0.26669
a1, ln(d)	0.14812	0.48747
a2, (d/10) ²	-0.08502	-0.16238
a3, ln(G)	-0.05519	-0.20152
a4, ln(SI)	0.24087	0.14162
a5, BAL/√(d+1)	-0.07529	-0.05902
a6, ln(t)	-0.43337	-0.43393

performance was also analysed by simulating stand development for all measurement intervals of all plots, then comparing measured periodical basal area increments to simulated increments (Table 4). In this comparison, the optimization-based diameter increment model obtained with loss function 1 resulted in the smallest bias and a RMSE slightly larger than the lowest RMSE (Table 4). The fixed-effects regression model underestimated basal area increment by 7.5%.

The simulations suggest that the optimization-based model obtained with loss function 1 might be the most appropriate for simulating stand development. The fixed-effects regression performed best in the tree-level evaluation. Therefore, the parameters of these two models are shown (Table 5).

Optimization examples

The models presented in this article can be used to simulate the development of Icelandic lodgepole pine plantations. However, as the model set does not include a survival function, only schedules in which the stand density is kept below the self-thinning limit should be simulated.

Together with the existing taper model

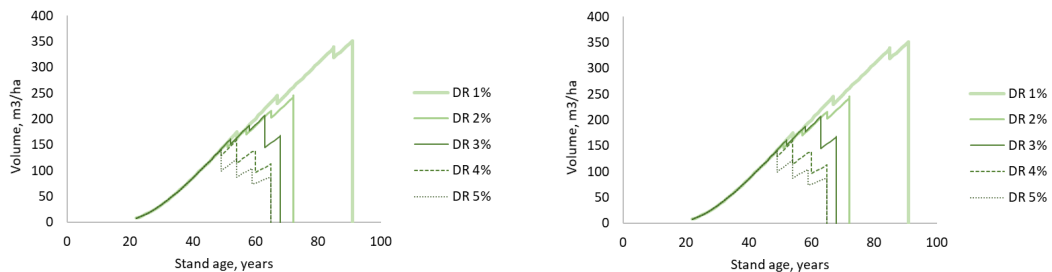
(Heidarsson & Pukkala 2011), the models allow the optimization of stand management. To illustrate the use of the models, we optimized the management of two plots of the dataset when the net present value (NPV) was maximized with different discount rates. All costs and incomes of the rotation were discounted to the planting year. It was assumed that the same rotation is repeated to infinity.

Of the two selected plots, plot 15 represents low site productivity, and plot 16 represents a productive site. The optimization-based diameter increment model obtained with loss function 1 was used.

As expected, the optimal rotation length increased with decreasing discount rate and site quality (Table 6, Fig. 5). The mean annual harvest increased with increasing rotation length, and the mean annual net income was also largest when low discount rate and long rotations were used (Table 6). Wood production (mean annual harvest) of the better site was about three times higher than the poorer site. On the poorer site, NPV was negative with discount rates of 4% and 5%, which meant that plantation forestry was not profitable on the poor site when the discount rate was high.

Table 6. Optimization results for two sample plots.

	Plot 15, site index 17.4 m				
	1%	2%	3%	4%	5%
Rotation length, years	91	72	68	65	65
Wood production, m ³ ha ⁻¹ a ⁻¹	4.11	3.57	3.35	2.96	2.69
Net income, IKR ha ⁻¹ a ⁻¹	76,000	66,000	61,000	53,000	47,000
NPV, IKR ha ⁻¹	4,496,000	1,139,000	296,000	-53,000	-210,000
	Plot 16, site index 24 m				
	1%	2%	3%	4%	5%
Rotation length, years	74	67	63	62	55
Wood production, m ³ ha ⁻¹ a ⁻¹	11.66	10.63	9.78	9.51	8.32
Net income, IKR ha ⁻¹ a ⁻¹	228,000	207,000	189,000	184,000	159,000
NPV, IKR ha ⁻¹	15,295,000	4,902,000	2,105,000	933,000	434,000

**Figure 5.** Development of growing stock volume on poor (top) and good (bottom) site when net present value is maximized with different discount rates (DR).

DISCUSSION

This study presents the first site index, individual tree diameter increments and height models for even-aged lodgepole pine stands in Iceland. Lodgepole pine has been cultivated on a larger scale in Iceland only for around 70 years, therefore the dataset did not contain information on older stands. This needs to be considered when making growth predictions for stands older than 70 years. The main part of the dataset represented stand ages between 10 and 40 years.

The height growth trajectories of the oldest Icelandic lodgepole pine stands do not show clear evidence of slowing height growth (Figure 3). On the other hand, the site index model suggests slower height growth at older ages and indicates that the maximum dominant height of

Icelandic lodgepole pine plantations is 38.8 m. The pattern is different than obtained recently for Sitka spruce (Heiðarsson et al. 2022). In Sitka spruce, there is no evidence of decreasing dominant height growth with increasing stand age (Fig. 6).

Of the few plantation species modelled so far in Iceland, Siberian larch (*Larix sibirica*) shows the fastest height growth at a young age and the slowest growth at older ages (Heiðarsson & Pukkala 2012). The shape of the site index model for lodgepole pine is between those of Sitka spruce and Siberian larch (Figure 6). However, none of the species has data from old stand ages, which means that all the models and curves shown in Figure 6 should be taken as preliminary. The models need to be checked

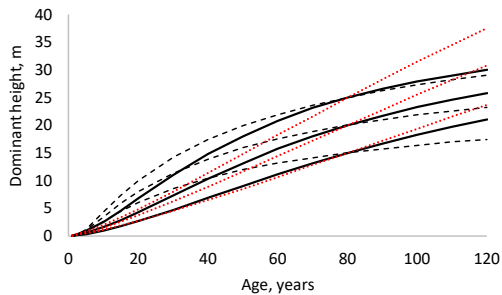


Figure 6. Dominant height development in site indices 15, 20 and 25 m according to the site index curves for larch (dashed black lines), Sitka spruce (red dotted lines) and lodgepole pine (black continuous lines).

and updated every few years, as the permanent sample plots in which growth is being monitored age. For lodgepole pine, the site index curves for stands younger than 10–15 years should also be used with caution because the height growth in younger stands is affected by factors other than site index (Borders et al. 1984, Barrio Anta & Dieguez-Aranda 2005).

The optimization-based approach (Pukkala et al. 2010) was used in diameter increment modelling because of irregular measurement intervals ranging from 3 to 14 years. Calculating annual increment by dividing periodical increment by the length of the period is equal to assuming a constant growth rate between two measurement occasions. This can lead to under- or over-estimation of tree growth when the growth dynamics are nonlinear (Clutter 1983, McDill & Amateis 1992, Cao 2000, Nord-Larsen 2006, Crecente-Campo et al. 2010). Previous studies have shown that, with regular data (constant measurement intervals), the optimization-based method produces very similar models to those obtained with regression analysis (Pukkala et al. 2011). In this study, the optimization-based diameter increment model obtained with loss function 1 resulted in the smallest bias in the simulation of the basal area increments of the plots. Because basal area correlates closely with volume, biomass, and carbon stock, our recommendation is to use the optimization-based model in long-term growth simulations.

According to Jin et al. (2019), a common problem in the growth modelling of plantation forests is the difficulty of modelling self-thinning. Often, the empirical data do not come from experiments that are designed for growth modelling (Jin et al. 2019). It may be that all plots represent normal planting densities and are regularly thinned, with the consequence that the sample plots used in modelling do not reach stocking densities where trees begin to die. This was the situation with Icelandic lodgepole pine plantations. The thinning treatments had most probably removed weak or suppressed trees that would otherwise have died soon. Therefore, the data available for growth modelling did not indicate stand conditions where self-thinning is likely to start. The consequence of this shortcoming in the available data is that the models cannot be used for such long-term simulations in which the stand is not thinned.

In the optimization examples provided in this study, it was assumed that the management schedule includes three thinning treatments during the rotation. Regular thinning of lodgepole plantations enhances root system development and improves the stability of the plantations. Thinning treatments decrease the risk of severe wind or snow damage. However, the thinning treatments should be light because otherwise the stands may be vulnerable to wind damage for a few years after the thinning (Valinger & Lundquist 1992). Therefore, even though the model system presented in this article lacks the mortality model, the models constitute a useful tool for supporting the management of lodgepole pine plantations in Iceland and the prediction of stand development.

The effects of discount rate and site productivity on optimal management were in line with previous studies, where optimal rotation lengths are shorter for good sites and high discount rates (e.g., Jin et al. 2017, de Miguel et al. 2014, Palahí & Pukkala 2003, Pasalodos-Tato & Pukkala 2007). At present, no thinning instructions exist for lodgepole pine in Iceland. The models of this study may help the development of management guidelines for different site indices.

There was a large difference in the economic profitability between the two site indices of the optimization examples. The mean annual harvest of the better site was about three times higher than that reached on the poorer site. On the poorer site, the NPV of the planting investment was negative with discount rates of 4% and 5%, which meant that plantation forestry was not profitable on the poorer site when the discount rate was high. In many cases, economic profitability can be expected to be lower than indicated by the optimization examples because the stem quality of first-rotation plantations in Iceland is often low due to the harsh climate (Birgisdóttir 2005). Often, trees get frost or snow damage, which decreases the wood quality at the lower parts of the stems. Consequently, the stem quality of first-rotation plantations is frequently insufficient for the most valuable sawlog products.

The size of the oldest lodgepole pine plantations of Iceland is often less than one hectare. To avoid edge effects in permanent sample plots, the plots are usually kept small. Small plot size leads to “sampling error” in some model predictors, for instance, in stand basal area and basal area in larger trees (BAL). In small plots, the surroundings may have a clear effect on the growth of the trees measured within the plots (Pukkala et al. 2013). If the stands are not homogeneous, the basal area of the plot may differ from the average basal area of the stand adjacent to the plot. In modelling, the competition was described by stand basal area and BAL within the plot. When these variables do not accurately reflect the true competition that the trees are facing, the modelled relationships between competition and tree growth may become “flatter” than the true competition effects.

To improve future modelling efforts, sample plots larger than the current ones should be established. It is also important to leave some of the plots unthinned, to provide information for self-thinning models. Since there is a lack of data from old stands, it is also necessary to continue the measurement of the current permanent plots. Because low stem and wood

quality are common problems in Icelandic lodgepole pine plantations, there is also a need to develop models for describing and predicting the technical quality of the stems of harvested trees. In addition, optimizing plantation management would also benefit from biomass modelling because an important role of Icelandic tree plantations is to sequester carbon from the atmosphere (Keller et al. 2022).

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