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Models for simulating the temporal development of black cottonwood (*Populus balsamifera* L. ssp. *trichocarpa* (Torr. & Gray ex Hook.) Brayshaw) plantations in Iceland

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ABSTRACT

Black cottonwood (*Populus balsamifera ssp. trichocarpa)* was initially introduced to Iceland in 1944 from the Kenai Peninsula in Alaska and has been widely planted in shelterbelts and afforestation projects since the 1980s. There is currently much interest in increasing the planting of black cottonwood, especially in carbon sequestration projects, because of its rapid growth at an early age. Growth models simulate the growth of a forest over time and are important tools for forest managers, researchers, and policymakers. This study presents, for the first time, site index, individual-tree diameter increment and tree height models for even-aged black cottonwood stands in Iceland. The data were collected from Icelandic national forest inventory (NFI) plots and from three plots from a network of permanent sample plots (PSP). The NFI data were collected during 2005–2022, and the PSP data were collected between 2009 and 2022. The model of McDill and Amateis was selected for predicting site index and dominant height development, and the model of Schumacher was selected for predicting tree height. For diameter increment modelling, an optimization-based modelling approach was found to be more suitable than non-linear regression analysis. The models developed in this study can be used in forestry practice and in optimization studies for thinned black cottonwood stands. The models produced simulation results that corresponded to measured stand development.

Keywords: Growth model, individual-tree model, optimization-based modelling, plantation forestry

YFIRLIT

Jöfnur sem lýsa vexti alaskaaspar (Populus balsamifera L. ssp. trichocarpa) *á íslandi.*

Alaskaösp var fyrst flutt til landsins frá Kenai í Alaska árið 1944 og hefur verið mikið notuð í skjólbelta- og skógrækt frá 1980. Í dag er mikill áhugi á aukinni notkun á alaskaösp, sérstaklega í kolefnisverkefnum vegna hraðs vaxtar snemma á æviskeiðinu. Vaxtarjöfnur spá fyrir um framtíðarvöxt skóga og eru mikilvæg verkfæri fyrir skógarstjórnendur, vísindamenn og stefnumótendur fyrir ákvarðanartöku, meðal annars í loftslagsmálum. Í þessari rannsókn eru birtar í fyrsta sinn jöfnur sem lýsa gróskustigi, þvermálsvexti og hæðarvexti trjáa fyrir jafnaldra alaskaasparskóga á Íslandi. Gögnin sem notuð voru í rannsókninni eru trjámælingar, aðalega frá Íslenskri skógarúttekt (ÍSÚ) en þrír af mæliflötunum eru fastir mælifletir (FMF). Gögnunum úr ÍSÚ var safnað á árabilinu 2005–2022 og gögnunum frá FMF var safnað á árabilinu 2009–2022. Til að spá fyrir um gróskustig og yfirhæðarvöxt skóga var valin aðlöguð jafna sem gerð var af McDill og Amateis og til að spá fyrir um hæðarvöxt stakra trjáa var valin aðlöguð jafna gerð af Schumacher. Til að spá fyrir um þvermálsvöxt trjáa var notuð bestunarnálgun (optimization approach) en hún gaf nákvæmari niðurstöðu en blönduð aðhvarfsgreining (mixed-effect modelling). Jöfnurnar sem aðlagaðar voru að íslenskum aðstæðum í þessari rannsókn má nota til áætlanagerðar og arðsemisútreikninga í grisjuðum alaskaasparskógum. Áætlaður vöxtur með jöfnunum er samsvarandi vexti viðkomandi skóga.

INTRODUCTION

Today, black cottonwood (*Populus balsamifera* L. ssp. *trichocarpa* (Torr. & Gray ex Hook.) Brayshaw) is an important tree species in Icelandic forestry, covering an area of 3900 ha, or 7% of the cultivated forest in Iceland (data from the Icelandic National Forest Inventory, NFI). The species was initially introduced to Iceland in 1944 from the Kenai Peninsula, Alaska (Bragason 1995). Black cottonwood has become an important urban tree in Iceland and has been widely planted in shelterbelts and afforestation projects since the 1980s (Óskarsson et al. 1990, Sigurdsson 2001a). In Iceland, there is currently much interest in increasing the planting of black cottonwood, especially in carbon sequestration projects, because of its rapid growth at an early age.

No growth models exist for black cottonwood in Iceland today, and scientific knowledge regarding its growth, yield and management is scant. The main reason for the lack of models is the young age of Icelandic black cottonwood plantations. A few recently published growth studies on black cottonwood focused on diameter growth, biomass and density (Eggertsson 2019, Mikaelsson 2011, Jóhannsdóttir 2012), or on fertilising and economic profit of short rotation forestry (Bogason et al. 2018).

In the last decade, the development of tree growth models has been ongoing in Iceland (Heiðarsson & Pukkala 2012, Heiðarsson et al. 2022, Heiðarsson et al. 2023). Growth models simulate the growth of a forest over time and are important tools for forest managers, researchers, and policymakers. They help to optimize forest management, such as thinning and harvesting, to maximize timber production or economic return while minimizing environmental impacts (Weiskittel 2014). Advanced models can be used to predict the effects of climate change on forest growth. Model integration makes it possible to develop forest management strategies that mitigate the harmful effects of climate change, while providing timber for forest industries and income for forest landowners (Heinonen et al. 2018, Trouillier et al. 2020).

Generally, a growth model refers to a system of equations that predict the growth and yield of a forest stand under a wide variety of conditions (Vanclay 1994). Growth models can be divided into three broad categories: stand-level models, individual-tree models, and diameter distribution models (Munro 1974). Stand-level models are developed using stand-level information (Curtis et al. 1981, Vanclay 1994), whereas individual-tree models predict individual tree growth or mortality (Clutter et al. 1983, Palahí & Pukkala, 2003). Diameter distribution models use statistical probability density functions to characterize the stand structure (Bailey & Dell 1973, Newton et al. 2005). Tree-level models are further classified as distance-dependent (spatial) or distance-independent (non-spatial) models.

The recent trend in Iceland has been to develop distance-independent individual tree models (Heiðarsson & Pukkala 2012, Heiðarsson et al. 2022, Heiðarsson et al. 2023). This type of model was targeted also in the current study because the available data contained no spatial information. The model set we developed for black cottonwood consists of a site index (SI) model (top height growth model), a tree height model, and an individual tree model for diameter increment. We provide below the rationale for the development of these three models.

For even-aged monocultures, SI models are the most common tools for estimating site productivity. SI is defined as the dominant height, i.e. the average height of the 100 largest trees per hectare, at a chosen reference age (Monserud 1984, Skovsgaard & Vancley 2008, Burkhart & Tomé 2012). For most tree species, the height growth of dominant and co-dominant trees in a stand is a stable predictor of site quality, because it is not much affected by stand density or thinning operations, assuming thinning from below (Cieszewski & Bella 1989, Skovsgaard & Vancley 2008, Weiskittel et al. 2009, Burkhart & Tomé 2012). SI models are widely used in forestry practice and research, due to the strong correlation between stand height and volume production (Vancley 1994, Skovsgaard & Vancley 2008).

Information on tree heights is essential in forest inventories for computing tree volumes. Tree height information is also needed in growth and yield simulators (Mehtätalo et al. 2015). Because field measurements of tree height are rather time-consuming and therefore expensive, many forest inventories use predictive models to get height estimates for the trees based on their diameter.

Tree diameter increment is an important metric for estimating wood production and can be easily measured in inventories. Stand management decisions, such as when and how much to thin a stand, rely heavily on variables derived from tree diameters. The development of models for diameter increment usually employs data from permanent plots, in which all trees have been remeasured at regular intervals (Juma et al. 2014).

To achieve the above, there is a reasonable number of repeated measurements available on black cottonwood plots for various regions of Iceland. The datasets currently available include mainly younger stands between 10 and 30 years in age. In these datasets, there is only one unthinned control plot and a few plots in which the planting density deviates from normal densities.

This study aimed to develop a set of models for site index, tree height and diameter increment to predict the yield of black cottonwood plantations in Iceland. Because of the young age of the measured stands, these models should be looked at as preliminary, their main purpose being growth estimation in young stands over a short period of time.

MATERIAL AND METHODS

Sample plot data

The data used for black cottonwood (*Populus balsamifera* L. ssp. *trichocarpa* (Torr. & Gray ex Hook.) Brayshaw) stands in this study were mainly collected from Icelandic national forest inventory plots (NFI). The NFI data are a statistical sample of all forested land areas in Iceland. Three plots of the dataset are permanent sample plots (PSP) established by Land and Forest Iceland for growth measurements. The NFI data were collected during 2005–2022 and the PSP data were collected between 2009 and 2022. The NFI data were collected from 14 permanent plots in 14 locations. The PSPs were measured in two locations (Figure 1). All plots are in planted, even-aged black cottonwood stands, established by Land and Forest Iceland. The NFI plots were remeasured with 5-year intervals and included 42 growth periods (Table 1). Two of the PSP plots were measured annually, and one had a 9-year interval between measurements.

The dataset covered different site types and growth conditions, mainly from young stands. All the locations have an oceanic climate with an annual precipitation (1964–1990) of 700–1200 mm and a mean annual temperature of 3.2–4.5°C (Vedurstofa Islands 2017). For the same period, the mean maximum daytime temperature during June–August was 12.9–13.6 °C (Vedurstofa Islands 2017). The range in plot elevation was between 10 and 140 m a.s.l.

The sample plots were circular, and the size of the plots varied between 0.01 and 0.02 ha. On every measurement occasion, the diameter at breast height (DBH, at 1.3 m) was measured on all trees that had reached that height. On some of the NFI plots, the total tree height was measured only on sample trees. The tree selection for height measurements was based on DBH, and the aim was to get heights from different DBH classes. Height was measured with a measuring pole for trees shorter than 4 m and with Vertex Laser VL5 and Laser Tech distance and height measurement instruments for taller trees. Because of the young age of the sampled forests, there was no mortality in the

Table 1. Mean, standard deviation (SD) and range of the main characteristics of the study material on black cottonwood in Iceland. N: number of observations; DBH: diameter at breast height; Growth: 5-year DBH growth; G: stand basal area; Age: stand age; Hdom: dominant height.

Variable	N	Mean	SD	Maximum	Minimum
DBH (cm)	813	6.15	4.86	39.0	0.0
Height (m)	737	4.74	2.88	23.4	0.31
Growth (cm)	813	2.81	1.73	10.4	0.1
G (m^2ha^{-1})	42	7.58	11.09	45.6	0.01
Age (years)	42	21.9	6.59	48.0	12.0
Hdom (m)	42	6.26	3.88	23.2	2.07
Growth periods	42	5.0	0	9.0	1.0
Stems per hectare	42	1524	928	4400	400

Figure 1. Geographical locations of the study sites in Iceland. The red dots present the NFI plots, and the yellow triangles are PSP plots.

dataset, and no attempt was made to model tree survival. All regression models were fitted with the R software, version 4.3.1 (Posit team 2023).

Site index modelling

Two datasets were tested in the site index modelling: one with only first and last height measurements and one with all height measurements. Several functions commonly used in the algebraic difference approach (ADA) were tested (Palahí et al. 2004). The tested functions were: Korf and Lundmark (Korf 1939), Schumacher (1939), Chapman-Richards (Richards 1959) and the model of McDill and Amateis (1992). All models predict the dominant height H_2, at a certain time point T_2, using current dominant height H_1 and current age T_1 as predictors:

$$H_2 = f(T_1, H_1, T_2) + \varepsilon \qquad (1)$$

When T_1 is replaced by index age and H_1 is replaced by site index (dominant height at index age), the model gives the dominant height at age T_2 for site index H_1. If H_1 is the measured dominant height at age T_1 and T_2 is the index age, the model gives the site index. The index age was taken as 50 years, which has been previously used for black cottonwood in Alaska (Shaw & Packee 1998). Therefore, the site index is defined to be the dominant height of the stand at the age of 50 years.

Of the tested models, McDill and Amateis (1992) was selected for predicting site index and dominant height development.

$$H_2 = \frac{a_0}{1-\left(1-\frac{a_0}{H_1}\right)\times\left(\frac{T_1}{T_2}\right)^{a_1}} + \varepsilon \qquad (2)$$

where H_1 and T_1 are, respectively, dominant height and stand age at the first measurement, H_2 and T_2 are the same variables at the second measurement, a_0 and a_1 are parameters to be estimated and ε is the random error term of the equation.

Tree height modelling

The number of height observations available for individual tree height modelling was 737. Based on the study of Mehtätalo et al. (2015), the following models were tested: Näslund (1937), Schumacher (1939) and Curtis (1967). These models were the best among the 28 datasets tested in that study. As the first step, fixed-effect models were fitted. Then the models were further developed by adding random plot factors to the fixed parameters and the models. The best combination of random plot factors was obtained by testing all possible combinations. Finally, the model of Schumacher (1939) was selected for predicting tree height.

$$h = 1.3 + (a_0 + a_1 H_{dom}) \times \exp\left[\frac{-\{b_0 + b_1 H_{dom}\}}{d}\right] + \varepsilon \tag{3}$$

where h is the tree height, H_{dom} is the dominant height, d is DBH, a_0, a_1, b_0, b_1 are fixed parameters to be estimated. The estimated parameters of the height model were modelled as a function of dominant height, which allowed the height curve to change along stand development.

Diameter increment modelling

Two different methods were tested to fit the models for diameter increment: non-linear regression analysis and the optimization-based approach suggested by Pukkala et al. (2011) and used earlier in Iceland by Heiðarsson et al. (2022) for Sitka spruce and Heiðarsson et al. (2023) for lodgepole pine. The model had to include at least one predictor for each of the following three influences: tree size, competition, and site productivity. Tree size was described by DBH and its transformations, and the site index was used to describe the effect of site productivity. To describe competition, stand basal area and basal area in trees larger than the subject tree were tested.

As the first step, regression analysis and mixed-effect modelling were used to search for the best transformations and combinations of predictors for the model. The following model turned out to be the most satisfactory:

$$i_{\mathrm{DBH}} = \exp\left(a_0 + a_1 \ln d + a_2\left(\frac{d}{10}\right)^2 + a_3 G + a_4 \ln SI + a_5\left(\frac{BAL}{\sqrt{d+1}}\right)\right) + \varepsilon \tag{4}$$

where i_{DBH} is the diameter increment (cm), d is DBH (cm), G is the stand basal area (m^{2}ha^{-1}), SI is the site index (m), and BAL is the basal area in larger trees than the subject tree (m^{2}ha^{-1}).

The predicted variable in regression analysis was a five-year diameter increment. Tests with the regression model suggested that the model may overestimate diameter increment in long-term simulations if the stand is not thinned. The probable reason for this outcome was that the modelled effect of increasing basal area on diameter increment was not strong enough, i.e. regression analysis resulted in a too flat relationship.

In two sample plots, tree diameters were measured annually over six years. These plots allowed us to see that the annual diameter increment may decrease substantially during a five-year measurement interval, most probably because of increased competition (Figure 2).

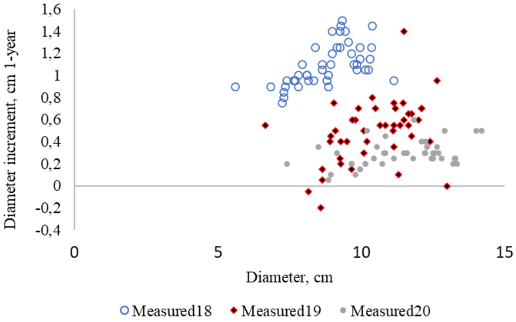

Figure 2. Relationship between diameter increment and tree diameter in the first, third, and fifth year of a five- year period in a plot where tree diameters were measured annually (at age 18, 19 and 20 years) for black cottonwood in Iceland.

To fully utilize the annually measured data from the two plots and to improve the modelled relationship between competition and diameter increment, the model was refitted using the optimization-based approach of Pukkala et al. (2011). In this model, the predicted variable of the model was annual diameter increment.

In the optimization-based approach, the tree diameters of the first measurement are used to start a simulation where tree growth is simulated from the first measurement to the second, using a one-year time step. The parameters of the diameter increment model are gradually adjusted, by using an optimization algorithm, so that the simulated diameter distribution at the end of the measurement interval corresponds to the measured diameter distribution of the trees.

The method minimizes a loss function, which describes the difference between the simulated and measured diameter distribution. The loss function used both the distribution of basal area and the distribution of the number of trees into different diameter classes. The minimized loss function was as follows:

$$\min z(\theta) = \sum_{k=1}^{K} \left[\sum_{j=1}^{J_k} w_{jk} \sum_{i=1}^{I_j} \left| g_{ijk}^{m} - g_{ijk}^{s}(\theta) \right| + 0.001 \left| n_{ijk}^{m} - n_{ijk}^{s}(\theta) \right| \right] \tag{5}$$

where Θ is the set of coefficients (a_0,… a_5 of Equation 4) estimated as arg min z(Θ), K is the number of plots, J_k is the number of measurement intervals of plot k, I_j is the number of 3-cm diameter classes in measurement interval j of plot k, g_{ijk}^{m} and $g_{ijk}^{s}(\Theta)$ are, respectively, measured and simulated cumulative basal area (m^{2}ha^{-1}) of diameter class i at the end of measurement interval j of plot k, and n_{ijk}^{m} and $n_{ijk}^{s}(\Theta)$ are, respectively, the measured and simulated cumulative number of trees per hectare in diameter class i at the end of measurement interval j of plot k (see, e.g., de-Miguel et al. 2014 for details). Symbol w_{jk} is a weight. The number of trees in plot k at the beginning of period j was used as the weight.

The optimization-based modelling does not produce direct information on the reliability of the coefficients. Therefore, bootstrapping (Varian 2005, Jin et al. 2019) was employed to find out how much the coefficients vary in repeated model fittings which are based on different samples. The model was fitted 30 times, using random sampling with replacement. The sample size was the same as the true number of measurement intervals, but the same measurement interval could be selected more than once, and some measurement intervals may not be selected for the sample.

RESULTS

Site index model

Of the tested site index models, the model of McDill and Amateis minimized the RMSE and the Akaike Information Criterion. Most of the models tested predicted the dominant height development similarly, but the behaviour of the selected model outside the age and dominant height range of modelling data was evaluated to be the most logical for the model of McDill and Amateis. Model versions based on all observations vs. the first and the last observation of each plot were almost identical (Figure 3). The model based on the first and last top height measurement is as follows:

$$\widehat{SI} = \frac{40.0182}{1-(1-40.0182/H_{\mathrm{dom}})} \times \left(\frac{T}{50}\right)^{1.4276} \tag{6}$$

Both parameters were significant at the 0.01 level. The coefficient of determination was 0.84 and the RMSE was 1.04 m.

When site index and stand age are known, the model can be used to calculate the dominant height for certain site index: H_{dom} is replaced by SI, stand age T is replaced by 50 (index age), and 50 is replaced by stand age:

$$\widehat{H_{\mathrm{dom}}} = \frac{40.0182}{1-(1-40.0182/SI)} \times \left(\frac{50}{T}\right)^{1.4276} \tag{7}$$

Figure 3 shows that the model followed the patterns of the measured dominant heights of the sample plots used in this study.

According to the model, the dominant height growth reached the maximum at different ages, depending on site productivity (Figure 4). For site index SI 25 and SI 20, the maximum was reached between 10 and 15 years, for site index SI 15 between 20 and 25 years and site index SI 10 between 30 to 35 years. At the age of 50 years, the annual dominant height growth was 0.3 meters or less in all site indices.

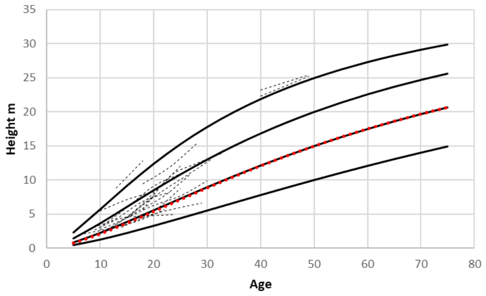

Figure 3. Dominant height curves for black cottonwood in Iceland (thick black lines) for site indices 10, 15, 20 and 25 m (site index = dominant height at 50 years) and the measured age and dominant height sequences of the study plots (thin dashed lines). The red dotted curve is the prediction for site index 15 based on the fixed part of a mixed-effects model that was fitted using all dominant height measurements of the dataset.

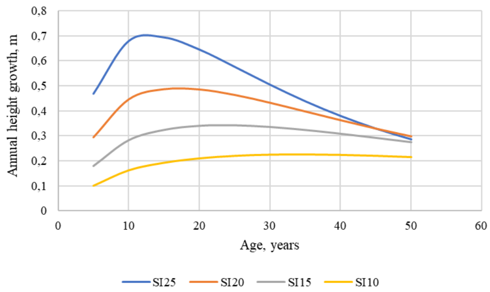

Figure 4. Annual height growth for different site indices (SI) for black cottonwood in Iceland. From above SI25, SI20, SI15 and at the bottom SI10 (site index = dominant height at 50 years).

Tree height model

The Schumacher model for tree height was as follows:

$$\hat{h} = 1.3 + (0.0543 + u_{0k}) \ + (1.1594 + u_{1k})H_{\text{dom}})$$

$$\times \exp\left[-\frac{(0.9747+0.4058H_{\text{dom}})}{d}\right] \tag{8}$$

where is the tree height, H_{dom} is the dominant height, d is DBH, and u_{0k}, and u_{1k} are random factors for plot k (Table 2).

The standard deviation of the residuals (RMSE) for the mixed-effect model (when the random effects are used in prediction) was 0.46 m (Table 2). When the tree height predictions were calculated with the fixed part of the mixed-effect model (assuming that the random effects are zero), the RMSE was 0.62 m. The bias of the fixed part of the mixed-effect model was –0.048 m, i.e. the model underestimated tree height on average by 4.8 cm, which is not substantial. All parameters except a_0 were significant at the 0.001 level, and the residuals were normally distributed with a constant variance at different diameters (Figure 5). Figure 6 shows that the tree diameter-height curve rose when the stand developed.

Table 2. Standard deviations and correlations of the random plot effects of the height model for black cottonwood in Iceland (Equation 8).

Standard deviations		Correlations	
u_{0k}	1.040	u_{0k}	u_{0k}
u_{1k}	0.103	u_{1k}	-0.579
Residual	0.455		

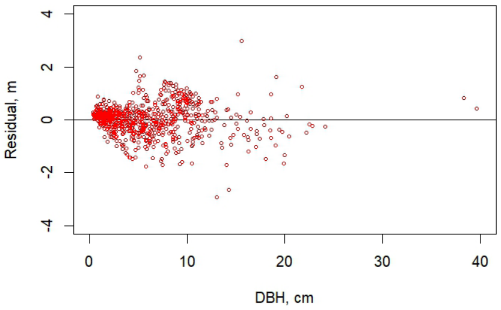

Figure 5. Residuals (observed-predicted) in predicting tree height with the fixed part of the mixed model for black cottonwood in Iceland.

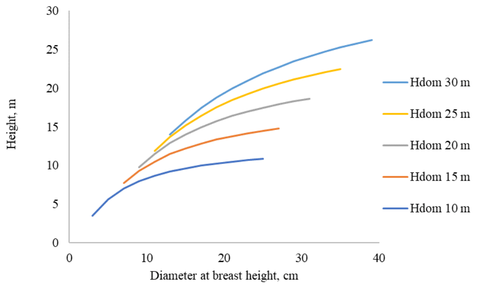

Figure 6. Relationship between diameter at breast height and tree height at different dominant heights (Hdom) for black cottonwood in Iceland.

Diameter increment model

The diameter increment model, fitted with the optimization-based approach, was as follows:

$$\widehat{\iota_{\mathrm{DBH}}} = \exp\left(-6.5902 + 0.5963\ln d - 0.1213\left(\frac{d}{10}\right)^2 - 0.0213G + 1.8793\ln SI - 0.1359\left(\frac{BAL}{\sqrt{d+1}}\right)\right) \quad (9)$$

where $\widehat{\iota_{\mathrm{DBH}}}$ is the future 1-year diameter increment (cm), d is the DBH (cm), G is the stand basal area (m^2ha^{-1}), SI is the site index (m) and BAL is the basal area in trees larger than the subject tree (m^2ha^{-1}). The bootstrap analysis suggested that all parameters of the model were significant (Table 3). The bias of the periodical basal area increment of the plot was 0.21 m^2ha^{-1}, which is 4.3% of the measured basal area increment. This means that the model slightly underestimated growth. The relative RMSE of the periodical plot-level basal area increment was 39%.

Figure 7 (top) shows the predicted diameter increment for different diameters and site indices when the stand basal area is constant and BAL decreases with increasing DBH. Figure 7 (top) indicates how trees of different DBHs would grow in an even-aged stand. The model predicted that the largest trees of the stand grow best, implying that the DBH differences between the smallest and the largest trees would increase with time.

Figure 7 (bottom) shows the effect of competition on diameter increment, with DBH set at 15 cm and BAL at 50% of the stand basal area. The diagram shows the strong negative effect of increasing stand density on DBH increment.

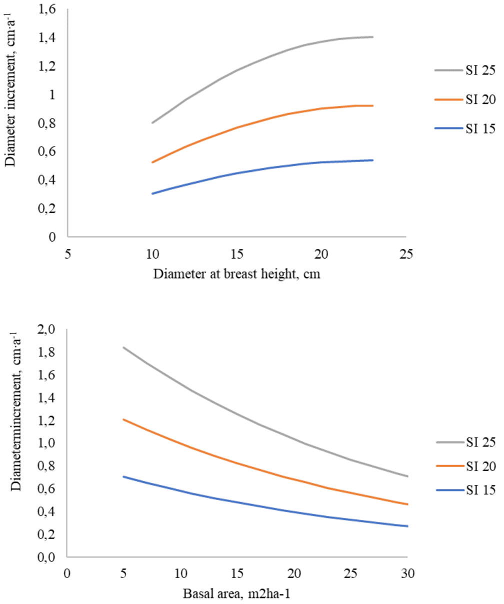

Figure 7. Top: Predicted diameter increment in an even-aged stand of black cottonwood in Iceland where DBH ranges from 10 to 23 cm, basal area is constant (15 m^2ha^{-1}) and BAL decreases from 15 to 0 m^2ha^{-1} when DBH increases from 10 to 23 cm. Bottom: Diameter increment with different stand basal areas when DBH is 15 cm and BAL is 50% of the stand basal area.

Simulation examples

The diameter increment model was used to simulate the development of four plots of the dataset in Icelandic black cottonwood plantations with different site indices and stand basal areas (Figure 8). In general, the models predicted basal area increments that were close to the measured basal areas (basal areas calculated from DBH measurements). However,

Table 3. Bootstrapping results for the significance of the coefficients of the optimization-based diameter increment model for black cottonwood in Iceland. The bootstrapping results are based on 30 model fittings using random sampling with replacement. Sdev is the standard deviation.

Parameter	a_0	a_1	a_2	a_3	a_4	a_5
Mean	-6.6767	0.5329	-0.10069	-0.02169	1.9454	-0.1330
Standard deviation	0.0993	0.0331	0.0110	0.0010	0.0374	0.0052
“t” Mean/Sdev	-67.23	16.10	-9.16	-22.49	52.08	-25.55

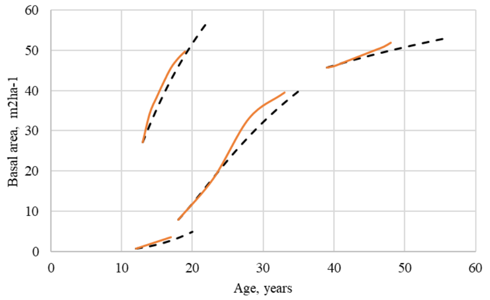

Figure 8. Examples of observed (continuous lines) and simulated (dashed lines) basal area development in four plots of the modelling dataset for black cottonwood in Iceland.

in one plot the model predicted slower growth rates than was measured. Also in this plot, the growth was predicted well for the first five years, from age 18 to age 23 years, but thereafter the measured growth was faster than the model prediction.

Residuals of observed vs. predicted values of DBH and BAL development from the two plots that were measured annually over six years are shown in Figure 9. The residuals of the model show that predictions for these two plots were unbiased, and there were no linear trends between the residuals and DBH or BAL. However, the scatter plots for DBH show a decreasing–increasing pattern, which may be explained, for example, by climate-induced annual variation in diameter increment. Figure 9 shows the residuals only for two out of 14 plots. When alternative models were tested with the full dataset, no transformations of DBH were found that resulted in better models than the one shown in Equation 9.

DISCUSSION

This study presents, for the first time, site index, individual tree diameter increment and tree height models for even-aged black cottonwood stands in Iceland. The available data for the growth modelling were mainly from NFI plots, which were not established for modelling purposes. As can be seen in Table 1 and Figure 3, the dataset is mainly from young stands between 10 and 30 years of age. The oldest stand was only 48 years old at the end of the measurement period. There was no mortality in the dataset, and data from very dense stands were lacking. The lack of mortality modelling limits simulations for stands older than 30 years and stands with high basal area (over 45 m^2ha^{-1}). Mikaelsson (2011) showed that survival rate is affected by high basal area.

The selection of the site index model was based on biological consistency, such as the value of the asymptote, on biological realism of the site index curves when compared with the modelling data, and on the behaviour of the model outside the age and dominant height range of modelling data (Figure 3). Figure 3 shows that the developed dominant height model resembles well the trends in the modelling data. The asymptote parameter of the dominant height model was 40 m, implying that the dominant height continues to grow at a rather

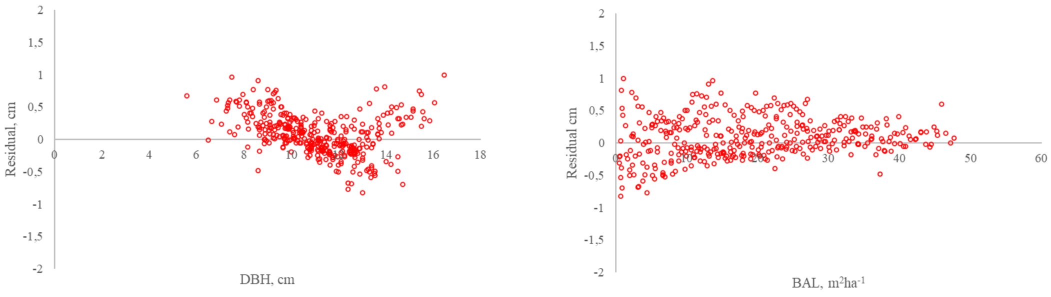

Figure 9. Residuals of the diameter increment model plotted against DBH and BAL in two plots where diameter increments were measured annually over six years.

old age. Forty meters is a realistic asymptote since black cottonwood trees can reach that height in favourable environments (Shaw & Packee 1998). In Iceland, the species is already approaching 30 m height at the oldest sites that were planted in the most favourable conditions (B.D. Sigurdsson, personal information). For stands younger than 10–20 years the site index curves should also be used with caution, because the height growth in younger stands is also affected by factors other than site index (Borders et al. 1984, Barrio Anta & Dieguez-Aranda 2005).

For site index SI 25 and SI 20, the maximum dominant height growth for black cottonwood occurs between 10 and 15 years (Figure 4). For site index SI 15, the maximum growth rate is reached between 20 and 25 years, and for site index SI 10 between 30 to 35 years. This is later than in Siberian larch (*Larix sibirica*), which is another pioneer species used in afforestation in Iceland (Heiðarsson & Pukkala 2012). At the age of 50 years, the annual dominant height growth rate of black cottonwood is 0.3 meters or less in all site indices, which is a realistic finding.

The tree height model is useful not only in yield simulators but also in predicting individual tree heights in field inventories when heights are not measured for all trees. As can be seen in Figure 5, the fit of the model is good, with no obvious trends or biases in the residuals. The selected tree height model guarantees that the simulated height development of individual trees is logically related to the dominant height development of the stand.

The height model was fitted as a mixed-effects model, which makes it possible to calibrate the model for a particular stand or plot. Simulations for volume development (not shown) suggested that, in most plots, the calibration had only a negligible effect on the simulation results, as compared to simulations where the random parameters were assumed to be zero. However, there were a few plots in which the full model provided better simulation results than the fixed part of the mixed-effect model. Therefore, model calibration is recommended whenever height measurements are available from the stand (Temesgen et al. 2008, de-Miguel et al. 2013).

The first step in diameter increment modelling was to search for the best transformation and combinations of predictors. Because the dataset has a hierarchical structure (correlations among observations), mixed-effect modelling was used for parameter estimation. When testing the mixed-effects model in longer-term simulations beyond the range of the data, the model seemed to result in overestimated basal area growth in dense unthinned stands. One reason for this outcome could be that the modelled effect of increasing basal area on diameter increment was not strong enough. Another reason was the lack of a mortality model. The problem of overestimated growth of unthinned stands was mitigated by using an optimization-based modelling approach (Pukkala et al. 2011). This approach was able to fully utilize the annual diameter measurements of some plots, which revealed the decreasing diameter increment with increasing stand basal area. Still, the models should be used with caution in denser stands. The optimization-based modelling approach can estimate also plot-specific coefficients to account for correlated observations (Juma et al. 2014), but they were considered unnecessary due to the preliminary nature of the models of this study.

Simulated basal area increments were compared to measured diameter increments in a few plots (Figure 8). The simulated increments were very close to the measured ones, with a small tendency of underestimation. One reason for the deviations seen in Figure 8 could be true changes in site index, which may be related, for example, to improved soil properties because of planted trees or to sheltering effects of a denser stand as it fills up the growing space. In the plot where the model started to underestimate growth after 10 years, the site index was 16.5 meters at the first measurement occasion, but 5 years later it was already 20.7. However, in simulations, the site index was kept constant. In Iceland, trees are planted in treeless lands, which are often used as pastures. Planted trees may have a favourable effect on the site productivity. For example, trees produce litter, which may improve the soil due to the increased content of organic matter. It is also possible that tree roots gradually reach nutrient-rich or moister soil layers. Such an effect on height growth has been shown to occur in initial spacing experiments in Iceland planted in treeless landscapes (Jóhannsdóttir 2012). Because of these unique conditions of the Icelandic tree plantations, the site index estimates should not be regarded as permanent descriptions of site productivity. They should be updated every few years.

Tree growth depends on environmental variables like climate and soil. Therefore, annual variation in environmental variables, such as temperature and precipitation, can alter annual growth rates, which is e.g. utilized by dendrochronology to derive past annual weather dynamics from tree-ring data (Eggertsson 2019). In Figure 9, the residuals from the two annually measured plots over six years are shown. The are no obvious biases in the residuals, but there might be systematic errors in some years, when the summer has been cold, dry, etc.

The dataset for growth modelling in this study had some limitations, which made the modelling more challenging and may also affect the model prediction. The data had insufficient representation of stands older than 30 years and of dense unthinned stands. Also, some parts of Iceland are not represented in the dataset. To improve future modelling efforts, it is necessary to continue the measurement of the current permanent plots, establish new plots in areas where no data are available and leave some of the plots unthinned to provide information for mortality models. Our results pave the way for further studies on optimizing plantation management for maximal yield, carbon sequestration or economic profitability, or for just evaluating alternative management regimes for black cottonwood.

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